

## EXERCISES: POPULATION GROWTH

- (1) Consider the ODE

$$x' = rx\left(1 - \frac{1}{k}x\right),$$

where  $r$  and  $k$  are constants, both bigger than 0. This is a simple model for a population growth.  $r$  is the growth rate, and  $k$  is the “capacity”.

- (a) Find all equilibria of the equation. For which values of  $r$  and  $k$  is there a positive equilibrium?
- (b) Classify the equilibria. For which values of  $r$  and  $k$  is 0 a stable equilibrium? (this means that for small initial values, the population dies out). For which values of  $r$  and  $k$  is 0 a source? (This means that the population never dies out).
- (2) Consider another simple model of two populations, one of which is hunting (e.g. foxes), and one of which is being hunted (e.g. rabbits). The system of ODEs is

$$\begin{aligned}x' &= rx - \alpha xy, \\y' &= -cx + \beta xy.\end{aligned}$$

Here,  $r, c, \alpha, \beta$  are positive constants.

- (a) Find the equilibria of this system and classify them.
- (b) Fix values of  $r, c, \alpha, \beta$  and look at the phase space picture of this system in matlab. Are there periodic solutions? Try several different values for  $r, c, \alpha, \beta$ . How does the behavior of the solutions change with the values of  $r, c, \alpha, \beta$ ?