SOME MORE "HAND EXERCISES" FOR CHAPTERS 1-3

(1) Solve the following systems of Linear equations using Gaussian elimination

(a)

$$x_1 + 2x_2 + 3x_3 = 1$$
$$2x_1 + 5x_2 + 8x_3 = 2$$

(b)

$$x_1 + x_2 + x_3 + x_4 = 1$$
$$2x_1 + x_2 + 2x_3 + x_4 = 3$$

(c) $Ax = b$ with

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 4 & 3 & 3 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

(d) $Ax = b$ with

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 5 & 5 & 5 \\ -1 & 2 & -1 & -4 \\ -3 & 0 & -5 & -10 \end{pmatrix}$$

and

$$b = \begin{pmatrix} -1 \\ 0 \\ 3 \\ 5 \end{pmatrix}$$

(2) Suppose that $ad - bc \neq 0$. Solve the System

$$ax_1 + bx_2 = e$$
$$cx_1 + dx_2 = f$$

(3) Solve the following system over the complex numbers

$$z_1 + iz_2 + (2-i)z_3 = 2+i$$
$$(1+i)z_1 + 0z_2 + 3iz_3 = 1+4i$$
$$-2z_1 + (1-i)z_2 + (1+2i)z_3 = i$$

(4) (a) Solve the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 5 \\ x_1 - x_2 \qquad &= -1 \end{aligned}$$

Denote the free parameter by t

(b) Solve the system

$$\begin{aligned} x_3 + x_1 + x_2 &= 5 \\ x_1 - x_2 &= -1 \end{aligned}$$

Denote the free parameter by s

(c) Show that the solution sets of the two systems are equal

(5) Calculate the product $A \cdot B$ for the matrices

$$A = \begin{pmatrix} 1 & 0 & 2 & 2 & 0 \\ 3 & 2 & 4 & 5 & 2 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

(6) Let $A = \text{diag}(a_1, a_2, \ldots, a_n)$ and $B = \text{diag}(b_1, b_2, \ldots, b_n)$ be diagonal matrices. What is their product?

(7) What can you say about the product of to matrices of the following type (here, $*$ are arbitrary, possibly different entries)

$$\begin{pmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix} \cdot \begin{pmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix}$$

(8) Calculate the inverse of the following matrices

(a)

$$\begin{pmatrix} 0 & 1 & -4 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & -2 & -1 \\ 1 & 0 & 1 \\ 1 & -2 & -1 \end{pmatrix}$$

(9) Which of the following maps are linear? If a map is linear, calculate the corresponding matrix.

(a) $\varphi_1 : \mathbb{R}^2 \to \mathbb{R}^3 \quad \varphi_1((x_1, x_2)) = (x_2, 0, 3x_1 - 2x_2)$

(b) $\varphi_2 : \mathbb{R}^2 \to \mathbb{R}^3 \quad \varphi_2((x_1, x_2)) = (x_2, 0, 3x_1 - 2)$

(c) $\varphi_3 : \mathbb{R}^3 \to \mathbb{R}^3 \quad \varphi_1((x_1, x_2, x_3)) = (x_1, x_2^2, x_2)$

(d) $\varphi_3 : \mathbb{R}^3 \to \mathbb{R}^3 \quad \varphi_1((x_1, x_2, x_3)) = (x_1, x_2^2, x_2)$

(10) Let $\varphi : \mathbb{R}^3 \to \mathbb{R}^2$ and $\psi : \mathbb{R}^2 \to \mathbb{R}^2$ be defined as follows: φ projects every point parallel to the x_3-axis to the x_1 - x_2 plane. The map ψ reflects every point at the diagonal $x_1 = x_2$.

(a) Show that the maps φ and ψ are linear.

(b) Find the matrices for φ, psi, and $\psi \circ \varphi$.