(1) Recall that an undirected graph $G = (V, E)$ is 3-colorable iff there is a map $f : V \rightarrow \{\text{red, blue yellow}\}$ such that no edge is assigned the same color to both its end points. Define $3\text{COL} = \{\langle G \rangle \mid G \text{ is a 3-colorable graph}\}$. Give an explicit reduction showing $3\text{COL} \leq_p 3\text{SAT}$.

(2) Let us say that the map $f$ defined in Problem 1 is a partial 3-coloring of $G = (V, E)$ if the domain of $f$ is a subset $V' \subseteq V$.

Define the NP problem Partial-3COL as follows:

Instance: $\langle G, f \rangle$ where $f$ is a partial 3-coloring of the graph $G$.

Question: Can $f$ be extended to a three-coloring of $G$?

Give an explicit reduction showing that Partial-3COL $\leq_p 3\text{COL}$.

(3) Consider the following decision problem:

Nice-SAT

Instance: $\langle \varphi \rangle$, where $\varphi$ is a CNF formula such that every clause either consists entirely of unnegated variables or entirely of negated variables.

Question: Is $\varphi$ satisfiable?

Show that Nice-SAT is NP-complete. (You may use the fact that 3SAT is NP-complete.)