MAT234: Review sheet for the final exam

Winter 2008, University of Toronto

1 Material covered

The final exam is cumulative. Emphasis will be put on chapters 5 and 10, but one should also expect questions from the material covered before the midterm, namely chapters 2,3 and 7. You are allowed to bring a formula sheet ($8\frac{1}{2} \times 11$ both sided). A list of formulas will also be provided on the exam.

2 preparation

One should go over the problems in the assignments and in the midterm, as well as the problems covered in class and in the tutorial. One may also look at the final exams of the previous years.

3 Formulas included in the exam

The following formula will be written in the exam:

- Integration by parts:
$$\int u dv = uv - \int v du$$
- Fourier series:
$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos\left(\frac{\pi m x}{L}\right) + b_m \sin\left(\frac{\pi m x}{L}\right)\right)$$
with
$$a_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{\pi m x}{L}\right) dx, \quad b_m = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{\pi m x}{L}\right) dx,$$
- Trigonometric identities:
$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$
$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

- Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

- Reduction of order: If $y_1(x)$ is a solution to the equation

$$y'' + py' + qy = 0,$$

then $y_2(x) = v(x)y_1(x)$ is another solution, where

$$v(x) = \int \frac{1}{y_1^2} \left(e^{-\int p dx}\right) dx.$$

4 More problems

The following problems are taken from the exam of last year:

1. The temperature distribution of an insulated bar of length $L = 1$ satisfies the initial value problem:

$$u_{xx} = u_t, u_x(0,t) = u_x(1,t) = 0, \quad u(x,0) = \cos(2\pi x).$$

Find the temperature distribution at later times.

2. A vibrating string of length $L = 1$ satisfies the boundary-value problem

$$u_{xx} = u_{tt}, \quad u(0,t) = u(1,t) = 0, \quad u(x,0) = \cos(2\pi x), \quad u_t(x,0) = 0.$$

Find the displacement $u(x,t)$ of the string.

3. (a) Let $f(x)$ be an odd periodic function of period 2π defined by

$$f(x) = \begin{cases} \frac{2x}{\pi} & \text{for } 0 \le x \le \frac{\pi}{2}, \\ \frac{2}{\pi}(\pi - x) & \text{for } \frac{\pi}{2} \le x \le \pi, \end{cases}$$

in the interval $[0, \pi]$. Show that its Fourier series is given by

$$\sum_{n=1}^{\infty} \frac{8}{\pi^2} \sin\left(\frac{\pi n}{2}\right) \sin(nx).$$

(b) The temperature of a rod of length $L = \pi$ satisfies the initial value problem

$$u_t = u_{xx}, \quad u(0,t) = u(\pi,t) = 0, \quad u(x,0) = f(x).$$

Find the temperature $u(x,t)$ of the rod.

(c) Solve the following Laplace equation with Dirichlet boundary conditions

$$\begin{aligned} &u_{xx} + u_{yy} = 0, && 0 < x < 2\pi,\ 0 < y < \pi, \\ &u(x,0) = 0, && u(x,\pi) = 0, \\ &u(0,y) = 0, && u(2\pi,y) = f(y). \end{aligned}$$