Name: ______________________ Recitation: Tu Th ID#: ______________

MAT303: Midterm II

Wednesday, November 16 2005

Problems:	1	2	3	4	Total
Points:					

There are four problems. Do all work on these pages. No Calculators, cell phones or notes may be used. However, a sheet of formulas ($8\frac{1}{2} \times 11$ recto-verso) is allowed. The point value (out of 100) of each problem is marked in the margin.

1.

(10pts) (a) Find a particular solution to the differential equation

$$\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 3x - 1$$

(5pts) (b) Show that $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ using the identity $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$.

(10pts) (c) Find a particular solution to the differential equation

$$\frac{d^2y}{dx^2} + y = \cos^2 x$$

2. In a damped forced oscillations system $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \cos \omega t$, the amplitude of the steady periodic solution $x_p(t) = C\cos(\omega t - \alpha)$ is given by

$$C(\omega) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

(5pts) (a) By analogy, in the RLC system $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E_0 \cos \omega t$, what is the amplitude $A(\omega)$ of the steady periodic solution $Q_p(t) = A\cos(\omega t - \alpha)$?

(10pts) (b) For which values of R (in terms of L and C) does the RLC system above exhibits a phenomenon of practical resonance for the amplitude of the steady periodic solution $Q_p(t)$?

(5pts) (c) In that case, what is the frequency of practical resonance in terms of R, L and C?

(5pts) (d) Is it the same as the frequency of practical resonance for the amplitude of the steady periodic current $I_p(t) = \frac{dQ_p}{dt}(t)$? Explain.

3. Consider the mass-spring-dashpot system satisfying the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0.$$

(5pts) (a) If $\vec{\mathbf{y}}(t) = \begin{pmatrix} x(t) \\ \frac{dx}{dt}(t) \end{pmatrix}$, find a 2×2 matrix $\mathbf{A}$ such that $\frac{d\vec{\mathbf{y}}}{dt} = \mathbf{A}\vec{\mathbf{y}}$.

(5pts) (b) What are the eigenvalues of $\mathbf{A}$?

(5pts) (c) For each eigenvalue of $\mathbf{A}$, find an associated (nonzero) eigenvector.

(5pts) (d) What is the general solution of the linear system $\frac{d\vec{\mathbf{y}}}{dt} = \mathbf{A}\vec{\mathbf{y}}$?

(5pts) (e) Is it critically damped, overdamped or underdamped? Justify your answer.

4. Consider the first order linear system

$$\frac{d\vec{\mathbf{x}}}{dt} = \mathbf{A}\vec{\mathbf{x}}, \quad \text{with } \mathbf{A} = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 4 \\ 0 & 4 & 0 \end{pmatrix}.$$

(5pts) (a) What are the eigenvalues of $\mathbf{A}$?

(10pts) (b) For each eigenvalue, find an associated (nonzero) eigenvector.

(10pts) (c) What is the solution if initially $\vec{\mathbf{x}}(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$?