

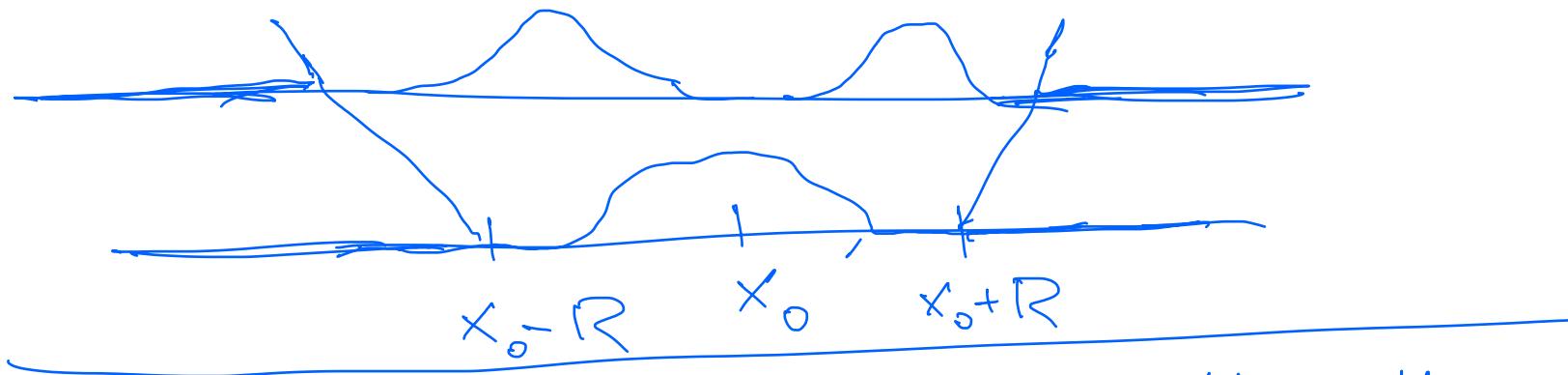
Comparison: Waves & Diffusion

	wave eqn $u_{tt} = c^2 u_{xx}$	diffusion eqn $u_t = u_{xx}$
(i) speed of propagation?	finite ($\leq c$)	infinite
(ii) Singularities for $t > 0$?	transported along	becomes immediately smooth
(iii) well-posed for $t > 0$?	yes	yes (at least for bounded ϕ)
(iv) well-posed for $t < 0$?	yes	no
(v) maximum principle?	no	yes
(vi) behavior for $t \rightarrow \infty$?	no decay	decays to zero/constant.

(i) wave eqn

$$u(x, 0) = 0 \text{ for } |x - x_0| \geq R$$

$$\Rightarrow u(x, t) = 0 \text{ for } |x - x_0| \geq R + ct$$



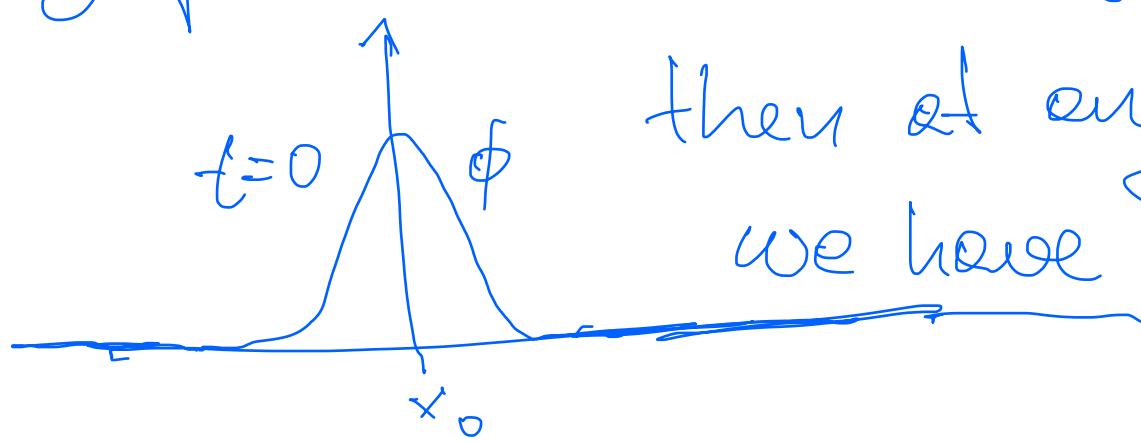
diffusion eqn

$$u_t = u_{xx}$$

$$u(x, 0) = \phi(x)$$

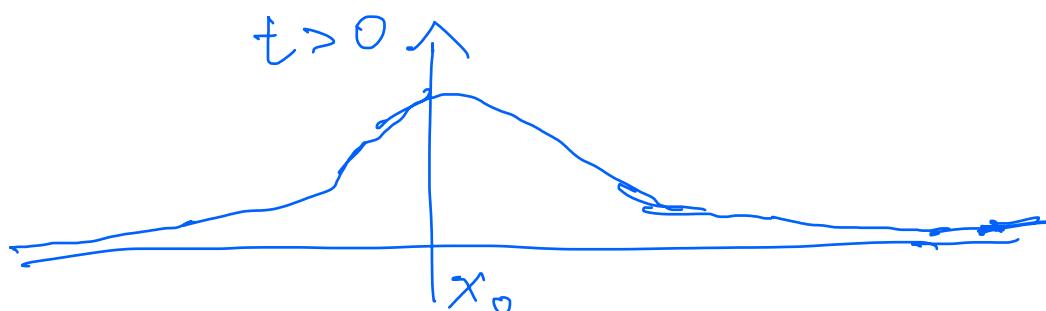
$$u(x, t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi t}} e^{-\frac{|x-y|^2}{4t}} \phi(y) dy$$

e.g. if $\phi \geq 0$, $\phi(x_0) > 0$ for some x_0 ,

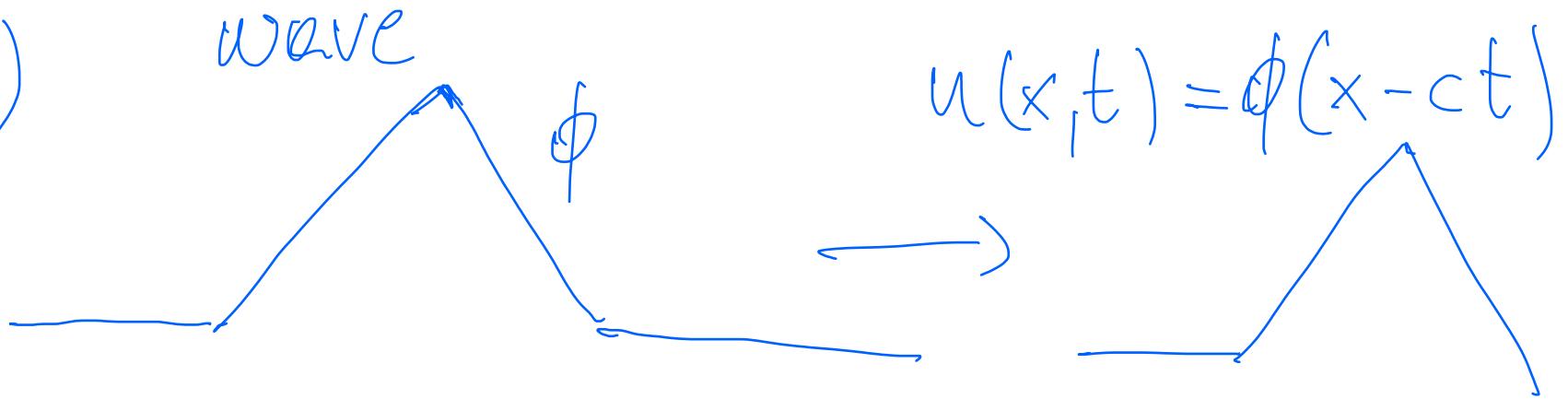


then at any $t > 0$
we have $u(x, t) > 0$

$$\forall x \in \mathbb{R}$$



(ii)



diffusion



(iii)

$$\left. \begin{array}{l} u_{tt} = c^2 u_{xx} \\ u(x,0) = \phi(x) \\ u_t(x,0) = \psi(x) \end{array} \right\} \Rightarrow u(x,t) = \frac{1}{2} [\phi(x+ct) + \phi(x-ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(s) ds$$

Existence, uniqueness & stability

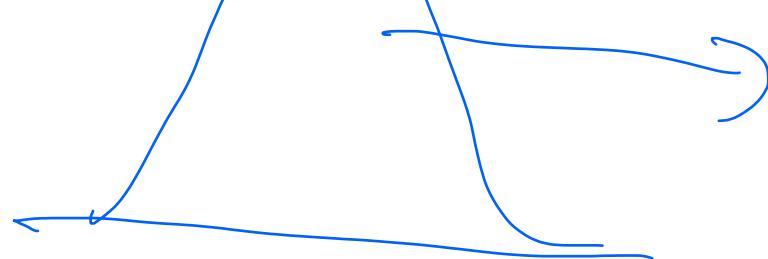
(iv) heat flow is an irreversible process!

(g) Ex

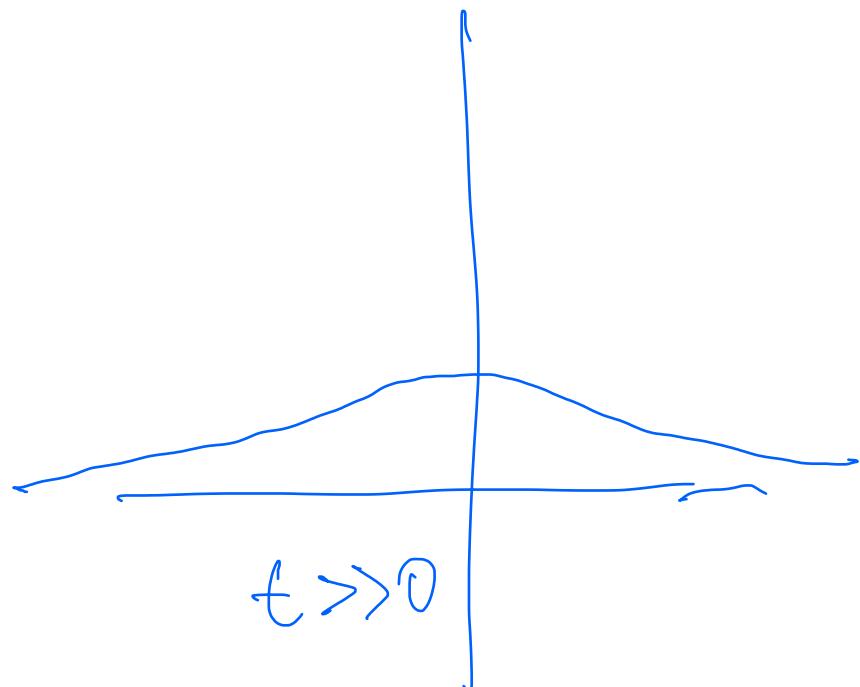
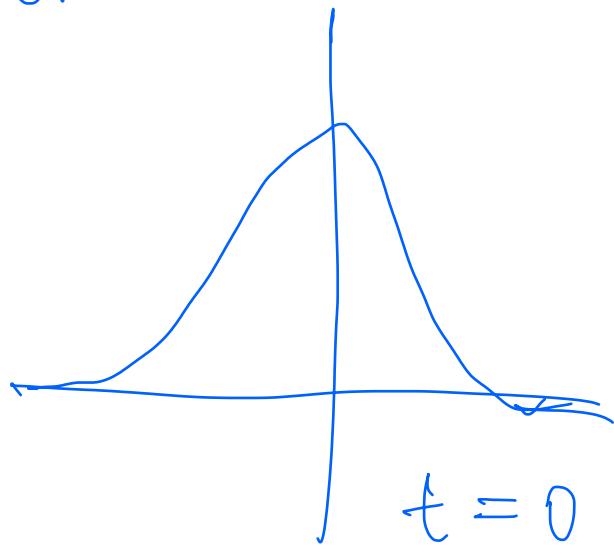


(vi) wave

$$u(x,t) = \phi(x - ct)$$



diffusion



little break.

Midterm 3 - 5

(submit via crowdmark)

