Partial Differential Equations (MAT D46), Final Exam 2021

1. True or false? Please simply answer with “true” or “false” (do *not* write a justification).
   i. The PDE $u_{xx} + u_{yy} = 0$ is linear.
   ii. Solutions to PDEs are always unique.
   iii. The function $u(x, y) = x^2 - y^2$ is harmonic.
   iv. Suppose $u_{tt} = 100u_{xx}$. If $u(x, 0) = 0$ for $|x| > 1$, then $u(x, 2) = 0$ for $|x| > 3$.

2. Solve the initial value problem $u_{tt} = 4u_{xx}$, $u(x, 0) = x^2$, $u_t(x, 0) = \cos(x)$.

3. Solve $u_{xx} + u_{yy} = 0$ in the disk \{r < 1\} with the boundary condition $u = 1 + 3\sin \theta$ on $r = 1$.

4. Solve $u_t = u_{xx}$, where $0 < x < \pi$, with the boundary conditions $u_x(0, t) = u_x(\pi, t) = 0$ and the initial condition $u(x, 0) = \cos^2 x$.

5. Consider the heat equation $u_t = u_{xx}$, where $-1 < x < 1$, with the boundary condition $u(-1, t) = u(1, t) = 0$ and the initial condition $u(x, 0) = 1 - x^2$. Show (a) that $u(x, t) = u(-x, t)$ for all $t \geq 0$ and all $-1 \leq x \leq 1$, and (b) that $0 < u(x, t) < 1$ for all $t > 0$ and all $-1 < x < 1$.

   To receive full credits, complete and clear computations and/or proofs must be provided (except for the true or false question).