UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCE II
TEST #3. MARCH 7, 2000

INSTRUCTIONS: Write your name and your student number on the front page of your examination booklet. This test consists of eight questions. The value of each question is indicated (in brackets) by the question number. Total marks: 100. Show all your work in all questions. Use both sides of the papers, if necessary. Do not tear out any pages. No calculators or any other aids are permitted. This test is worth 20% of your course grade. Duration: 2 hours.

1. (15 marks) Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 2$.

2. (15 marks) A lamina occupies the region $D = \{ (x, y) \mid -\pi/2 \le x \le \pi/2 \ , \ 0 \le y \le \cos x \}$ and has density function $\rho(x, y) = y$. Find the coordinates of the centre of mass of this lamina.

3. (15 marks) Evaluate $\iiint\limits_R z\,dV$, where R is the solid region in the first octant that lies between the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2 + y^2}$.

4. (15 marks) Use the transformation $u = x + y$, $v = x - y$ to evaluate the integral $\iint\limits_T (x + y)^{-2}\,dA$, where T is the trapezoidal region with vertices $(1, 1)$, $(3, 3)$, $(6, 0)$, and $(2, 0)$.

5. (10 marks) Let C be the curve of intersection of the surfaces $z = x^2 + y^2$ and $x = 2y$. Determine the work done by the force $\mathbf{F}(x, y, z) = (x - z)\,\mathbf{i} + (1 - z)\,\mathbf{j} + y\,\mathbf{k}$ on a particle that moves along the curve C from $(0, 0, 0)$ to $(2, 1, 5)$.

6. (10 marks) Show that the line integral $\int_C (y^2\cos(xy)\,dx + (\sin(xy) + xy\cos(xy))\,dy$ is independent of path and evaluate it over any path from $(\pi, 1/2)$ to $(\pi/2, 3)$.

7. (10 marks) Use Green's Theorem to evaluate the line integral $\int_C (e^x - xy)\,dx + (x^2 + \ln(1 + y))\,dy$, where C is the triangle with vertices $(0, 0)$, $(1, 2)$, and $(0, 3)$, positively oriented.

8. a) (5 marks) Let $\mathbf{F}(x, y, z) = (ay^3 - z^2)\,\mathbf{i} + (bz + xy^2)\,\mathbf{j} + (cxz + 3y)\,\mathbf{k}$, where a, b, and c are constants. Compute the curl of $\mathbf{F}$ and find values of a, b, and c, if any, for which $\mathbf{F}$ is conservative.
   b) (5 marks) Is there a vector field $\mathbf{G}$ on R^3 such that $\operatorname{curl}\mathbf{G} = x^3\,\mathbf{i} + (z - 2x^2y)\,\mathbf{j} + (2 + z - x^2z)\,\mathbf{k}$? Justify your answer.