UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS

MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCE II
TEST #3. MARCH 9, 1999

INSTRUCTIONS: Write your name and your student number on the front page of your examination booklet. This test consists of six questions. The value of each question is indicated (in brackets) by the question number. Total marks:100. Show all your work in all questions. Use both sides of the papers, if necessary. Do not tear out any pages. No calculators or any other aids are permitted. This test is worth 20% of your course grade. Duration: 2 hours.

1. a) (7 marks) Let $\mathbf{F}(x, y, z) = (1 - xyz, 4y + y^2 z, x^2 - z)$ and $\mathbf{G}(x, y, z) = (yz, z, xy)$. Calculate div $\mathbf{F}$ and curl $\mathbf{G}$. Use these results to simplify the expression div $\mathbf{F} + \mathbf{G} \cdot$ curl $\mathbf{G}$.

   b) (8 marks) Find the second-degree Taylor polynomial for the function $f(x, y) = e^{3y} - x^2 + \frac{e^y}{x}$ at the point $\mathbf{a} = (1, 0)$.

2. (15 marks) Find all of the critical points of the function $f(x,y) = 5 + 3y + xy - y^2 - \frac{x^3}{3}$. Use the Second Derivative Test to identify each of them as a local minimum, a local maximum or a saddle point.

3. a) (10 marks) Compute the length of the path parametrized by $\mathbf{f}(t) = (\frac{1}{3}t^{\frac{3}{2}}, \sin t + \cos t, \sin t - \cos t)$, where $1 \le t \le 8$.

   b) (10 marks) Compute the area of the surface parametrized by $\mathbf{g}(s,t) = (s^2, \frac{1}{2}t^2, st)$, where $0 \le s \le 1$ and $0 \le t \le 2$.

4. (15 marks) Evaluate $\iiint_S \frac{1}{2+x}\,dV$, where S is the solid in the first octant bounded by the coordinate planes, the plane $x + 2y = 2$ and the parabolic cylinder $x^2 + z = 4$.

5. (15 marks) Evaluate the line integral of the vector field $\mathbf{F}(x, y) = (x, xy)$ over the path C, where the given path C consists of the portion of the unit circle $x^2 + y^2 = 1$ in the first quadrant that goes from $(1, 0)$ to $(0, 1)$, followed by the line segment that goes from $(0, 1)$ to $(2, 2)$.

6. a) (10 marks) Evaluate $\int_0^{\pi/8} \int_{2x}^{\pi/4} \frac{\sin^2 y}{y}\,dy\,dx$.

   b) (10 marks) Evaluate $\iint_R (\frac{y}{x^2 + y^2})^3\,dA$, where R is the region described by the conditions $0 \le x \le y$ and $x^2 + y^2 \ge 9$.