UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS

MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCE II
TEST #3. MARCH 3, 1998

NAME: _____ , _____ **STUDENT No.:** _____

(Family name. Please PRINT.) **(Given name.)**

INSTRUCTIONS: This test consists of eight questions. The value of each question is indicated (in brackets) by the question number. Total marks: 100. Show all your work in all questions. Give your answers in the space provided. Use both sides of paper, if necessary. Do not tear out any pages. Calculators or any other aids are not permitted. This test is worth 20% of your course grade. DURATION: 2 HOURS.

1. (10 marks) Evaluate the double integral $\iint_R \frac{x}{(x^2+y^2)^{3/2}}\,dA$,

where $R = \{(x,y)\colon 1 \le x^2+y^2 \le 9,\ 0 \le y \le x\}$.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	VALUE	GRADE
1	10	
2	10	
3	10	
4	15	
5	15	
6	15	
7	10	
8	15	
TOTAL	100	

2. (10 marks) Compute the surface area of the portion of the graph of the function $z = 7 + x\sqrt{3} + y^2$ that lies above the triangle with vertices $(0, 0)$, $(0, 2)$, and $(1, 2)$.

3. (10 marks) Calculate the volume of the solid region in the first octant $(x \geq 0 , \ y \geq 0 , \ z \geq 0)$, given by the conditions: $x + z \leq 4$ and $y^2 + z \leq 4$.

4. (15 marks) Evaluate the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$.

5. (15 marks) Determine the coordinates of the centroid of the region $x^2+y^2 \le z \le \sqrt{4-3x^2-3y^2}$, assuming that the density is constant.

6. (15 marks) Evaluate $\iint_R e^{(y-x)/(y+x)}\, dA$, where R is the region $x \geq 0$, $y \geq 0$, $1 \leq x + y \leq 2$.

Hint: Use the substitution $u = y - x$, $v = y + x$.

7. (10 marks) Suppose that it is known that the vector field $\mathbf{F}(x, y, z) = (3x^2y, x^3 + kyz, 5 - 3y^2)$ is the gradient of some scalar function $f(x, y, z)$.

a) Determine the value of the constant k and find a potential function f.

b) Evaluate $\int_{(0,1,-1)}^{(1,0,1)} \mathbf{F} \cdot d\mathbf{r}$.

8. (15 marks) Let the curve C be the ellipse with centre at the origin, a semi-axis of length 4 along the x-axis and a semi-axis of length 3 along the y-axis.

a) Use a line integral to calculate the work done by the force $\mathbf{F}(x, y) = (3x - 4y, 4x + 2y)$ in moving a particle counterclockwise, once around the curve C.

b) Double-check your answer to part (a) by applying Green's theorem to the problem.