MAT 235 TEST #1 MODEL SOLUTIONS

(5) $r(t) = \int r'(t)\,dt = \left(\frac{1}{4}t^2 + c_1,\ -\frac{4}{t} + c_2,\ 4\sqrt{t} + c_3\right)$

For $r(4) = (8, 8, 8)$.

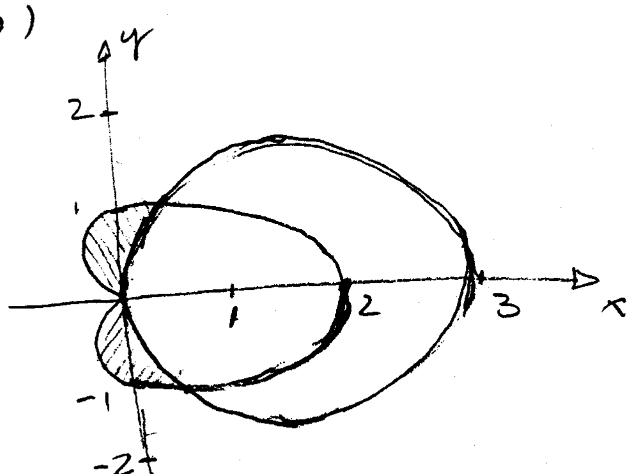
$4 + c_1 = 8$ $\qquad c_1 = 4$

$-1 + c_2 = 8$ $\qquad c_2 = 9$

$8 + c_3 = 8$ $\qquad c_3 = 0$

$r(2) = (1+4,\ -2+9,\ 4\sqrt{2}+0)$

$\boxed{r(2) = (5, 7, 4\sqrt{2})}$



(6) At the intersection points

$3\cos\theta = 1 + \cos\theta \qquad \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \quad \theta = \frac{5\pi}{3}$

$$A = 2\left[\int_{\pi/3}^{\pi} \frac{1}{2}(1+\cos\theta)^2\,d\theta - \int_{\pi/3}^{\pi/2} \frac{1}{2}(3\cos\theta)^2\,d\theta\right]$$

$$= \int_{\pi/3}^{\pi} (1 + 2\cos\theta + \cos^2\theta)\,d\theta - \int_{\pi/3}^{\pi/2} 9\cos^2\theta\,d\theta$$

$$= \int_{\pi/3}^{\pi} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta)\right)d\theta - \int_{\pi/3}^{\pi/2} \left[\frac{9}{2} + \frac{9}{2}\cos(2\theta)\right]d\theta$$

$$= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin(2\theta)\right]_{\pi/3}^{\pi} - \left[\frac{9}{2}\theta + \frac{9}{4}\sin(2\theta)\right]_{\pi/3}^{\pi/2}$$

$$= \left(\pi - \sqrt{3} - \frac{\sqrt{3}}{8}\right) - \left(\frac{3\pi}{4} - \frac{9\sqrt{3}}{8}\right)$$

$$= \boxed{\frac{\pi}{4}}$$