MAT 235 TEST #1 MODEL SOLUTIONS

① a) $m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-12+6t^2}{2t}$

For $m=3$: $6t^2-6t-12=0$ $6(t-2)(t+1)=0$

So, $t=-1$ or $t=2$.

The points are: $\boxed{(4,15) \text{ and } (7,-3)}$

b) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(dy/dx\right)}{dx/dt} = \frac{\frac{d}{dt}\left(-\frac{6}{t}+3t\right)}{2t} = \frac{\frac{6}{t^2}+3}{2t}$

At $(7,13)$: $t=-2$ and $\frac{d^2y}{dx^2}<0$

The curve is concave down

② $l = \int_1^2 \sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\, dt$

$= \int_1^2 \sqrt{(6t)^2+(3t^2+3)^2+(3t^2-3)^2}\, dt$

$= \int_1^2 \sqrt{18t^4+36t^2+18}\, dt$

$= \int_1^2 3\sqrt{2}\,(t^2+1)\, dt$

$= 3\sqrt{2}\left[\frac{1}{3}t^3+t\right]_1^2 = 3\sqrt{2}\left[\frac{7}{3}+1\right]$

$= \boxed{10\sqrt{2}}$