6. 1999-2000 TEST # 1 QUESTIONS.
1. a) (10 marks) Consider the curve C with parametric equations $x = t3 - 3t^2 + 1$ and $y = t3 + 2t - 5$. Determine the coordinates of all points on C , if any, where the tangent line is horizontal. Determine the coordinates of all points on C , if any, where the tangent line is vertical.
   b) (15 marks) Consider the curve with parametric equations $x = t2 - 2 \ln t$ and $y = 4t$,where $1 <\_ t <\_ 2$. Compute the area of the surface obtained by rotating this curve about the x-axis.
2. a) (10 marks) Compute the length of the polar curve $r = \sin2 (0/2)$, where $0 < 0 <\_\_$ 7c.
   b) (15 marks) Consider the curves given by the polar equations $r = 2 \sin 0$, and $r2 = 2 \cos (20)$. Compute the area of the region that lies inside both polar curves.
3. a) (8 marks) A conic is given by the polar equation $r = 2 J (1 + \cos 0)$. Find its eccentricity, identify the conic, locate the directrix, give a Cartesian equation of the curve and roughly sketch the conic.
   b) (7 marks) Identify the quadric surface given by the rectangular equation $x2 + z2 = 3 + 2y - y2$. Give both, an equation in cylindrical coordinates and an equation in spherical coordinates for this surface.
4. Consider the points A(1, 1,0), B(2, 1,-1), C(0,3,2),and P(1,3,k)
   a) (5 marks) Determine all values of the constant k, if any, for which the angle between the vectors AB and AP is 7r / 3 .
   b) (10 marks) Determine all values of the constant k, if any, for which the distance from the point P to the plane that passes through A, B , and C is equal to 4 .
5. a) (5 marks) Find an equation for the plane consisting of all points that are equidistant from (4 , 3 , - 3) and(-2,3, 1).
   b) (5 marks) Let L denote the line that passes through the points (1 , 1 , 1) and (- 1 , 2 , 2). Find parametric equations for the line L . Does the quadric surface $x2 + y2 = I + z2$ contain all the points of the line L ? Why or why not?
6. (10 marks) Select and answer only one of the following two questions 6.1 or 6.2.

   6.1. Consider the curve given by the polar equation $r = 1 + J0 \int_o \frac{1}{3+\cos t} dt$. Determine the slope of the tangent line to the curve at the point where $0 = 0$.
   6.2. Let a, b, and u be three vectors such that $\| a \| \neq 0$, $11 b ll = 2 \| a I)$, $a \times u = b$, and $a - u = h a \|$. Compute $\| u \|$.
Note: The topics for this 1999-2000 term test # 1 did not include Chapter 14.