UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS

MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCES
TEST #1, OCTOBER 16, 1996

NAME: _____ , _____ STUDENT No.: _____
(Family name. Please PRINT.) (Given name.)

INSTRUCTIONS: This test consists of six questions. The value of each question is indicated (in brackets) by the question number. Total marks: 80. Show all your work in all questions. Give your answers in the space provided. Use both sides of the paper, if necessary. Do not tear out any pages. No calculators or any other aids are permitted. This test is worth 20% of your course grade. Time allowed: 2 hours.

1. Given the vectors $\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (0, -1, 1)$, and $\mathbf{w} = (-3, 2, 0)$.
   a) (5 marks) Compute $\| \mathbf{w} - 3(\mathbf{u} + 2\mathbf{v}) \|$.
   b) (5 marks) Determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.
   c) (5 marks) Determine all values of k, if any, for which the vector $\mathbf{u} + k\mathbf{w}$ is perpendicular to the vector $\mathbf{v} \times (\mathbf{u} - \mathbf{w})$.

2. Given the points $A = (2, 0, -1)$, $B = (1, 4, 0)$, and $C = (-3, 2, 0)$.
   a) (5 marks) Compute the area of the triangle whose vertices are the points A, B, and C.
   b) (5 marks) Find an equation of the plane that contains all three points A, B, and C.
   c) (5 marks) Find parametric equations of the line that passes through the point A and is parallel to the line BC.

3. a) (5 marks) Find the value of a, for which the lines $(x, y, z) = (-1 + 2t, -6 + t, 5 - t)$ and $\frac{x-1}{-2} = \frac{y+1}{1} = \frac{z+a}{3}$ intersect, and determine the coordinates of the intersection point.
   b) (5 marks) Find the coordinates of the point of the line $(x, y, z) = (4 - t, t, 9 + 2t)$ which is closest to the point $(0, 0, 1)$.
   c) (5 marks) Let L denote the line of intersection of the planes $2x - y + z = 0$ and $x - 3z = 6$. Find an equation of the plane that contains the line L and is parallel to the line $x = y = z$.

4. a) (5 marks) Give the equation in rectangular coordinates, the equation in cylindrical coordinates, and the equation in spherical coordinates of the sphere with center at $(0, 0, 3)$ and with radius 2.

   b) (5 marks) Find parametric equations for the curve of intersection of the surfaces $y^2 + z^2 = 4$, and $x + 3y^2 + 3z^2 = 13$.

   c) (5 marks) Given the quadric surface $x^2 - 3y^2 + z = 0$, identify the level curves when $z < 0$, when $z = 0$, and $z > 0$. Identify the curves of intersection with the xz-plane, with the yz-plane, and briefly describe the surface.

5. Given the curve $r(t) = (\sin^3 t, \frac{4}{3}\sin^3 t, \frac{5}{3}\cos^3 t)$.

   a) (5 marks) Compute the arclength of the curve from $t = \frac{\pi}{6}$ to $t = \frac{\pi}{4}$.

   b) (5 marks) Determine the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$ at $t = \frac{\pi}{3}$.

6. (10 marks) Determine the coordinates of the point on the curve $\mathbf{r}(t) = (t^2 + 2t, t^2 + 3, 1 - t)$ at which the curvature has its maximum value.