UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCE II
TEST #3. MARCH 5, 2002

INSTRUCTIONS: Show all your work in all questions. Use both sides of the papers, if necessary. Do not tear out any pages. Do not use pencils. Only pen written answers will be considered for remarking. No calculators or any other aids are permitted. Write your name and your student number on the front page of each of your examination booklets. This test is worth 20% of your course grade. Duration: 2 hours.

1. a) (10 marks) Compute the surface area of the part of the cone $z^2 = 4\,(x^2 + y^2)$ that lies between the planes $z = 2$ and $z = 4$.

   b) (15 marks) Evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 \, dz\, dy\, dx$.

2. a) (10 marks) Evaluate the line integral $\int_C z\, ds$, where C is the curve given by the parametrization $x = 2t$, $y = t^3/3$, $z = 2t^3/3$, $0 \le t \le 1$.

   b) (10 marks) Use Green's Theorem to evaluate the line integral $\int_C (y^2 + \sin(x^2))\, dx + (x + \cos(y^2))\, dy$, where C is the triangular curve consisting of the line segments from $(0, 0)$ to $(1, 0)$, from $(1, 0)$ to $(1, 1)$, and from $(1, 1)$ to $(0, 0)$.

3. a) (10 marks) Let $\mathbf{F}(x, y, z) = x\,\mathbf{i} + z\,\mathbf{j} + a\,y\,\mathbf{k}$. Find all the values of the constant a, if any, for which $\operatorname{div}(\mathbf{F} \times \operatorname{curl} \mathbf{F}) = \operatorname{div} \mathbf{F}$.

   b) (15 marks) Let $\mathbf{G}(x, y) = e^{-2y} \sin x\, \mathbf{i} + (2\,e^{2y} + 2\,e^{-2y} \cos x)\, \mathbf{j}$. Find a function $g(x, y)$ such that $\nabla g = \mathbf{G}$, and use it to evaluate the line integral $\int_C \mathbf{G} \cdot d\mathbf{r}$, where C is the arc of the curve $y = \cos^3 x$, from $x = 0$ to $x = \pi$.

4. (15 marks) Compute the mass of the solid in the first octant, bounded by the cylinder $y^2 + z^2 = 1$, and the planes $x = 0$, $y = 0$, $z = 0$ and $x + y = 2$, if the density function is $\delta(x, y, z) = 2z/(1 + y)$.

5. (15 marks) Evaluate $\iint_R (x + y)^2\, dA$, where R is the region bounded by the curves $x + y = 2$, $x + y = 4$, $x^2 - y^2 = 4$ and $y = x$. (Hint: Use an appropriate change of variables.)