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UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCE II
TEST #2. DECEMBER 14, 2001

INSTRUCTIONS: Show all your work in all questions. Use both sides of the papers, if necessary. Do not tear out any pages. Do not use pencils. Only pen written answers will be considered for remarking. No calculators or any other aids are permitted. Write your name and your student number on the front page of each of your examination booklets. This test is worth 20% of your course grade. Duration: 2 hours.

1. a) (10 marks) Find $\partial z / \partial x$ and $\partial z / \partial y$ if $x y \sin(y+z) + e^{xz} = 1$.

   b) (10 marks) Let $f(x,y) = (x+3y)^4$, where $x = u^2 v$ and $y = 2u - v$. Use the Chain Rule to evaluate $\partial f / \partial v$ when $u = -1$ and $v = 2$.

2. a) (10 marks) Use the linear approximation of the function $f(x,y) = e^{x-2y}$ at the point $(2,1)$ to approximate the value of $f(2.01, 1.02)$.

   b) (10 marks) Let $f(x,y,z) = x^2 y - y^3 z + z^2$. Find the maximum possible value of the directional derivative $D_{\mathbf{u}} f(x,y,z)$ at the point $P(1,1,1)$.

3. (15 marks) Let $u = (x^2+y^2)^{3/2}$. Find the value of the constant k for which $u_{xx} + u_{yy} = k\, u^{1/3}$.

4. a) (15 marks) Find all critical points of the function $f(x,y) = x^3 - 15x + 3xy^2 + y^3$, and use the second derivative test to classify each of these critical points as a local minimum, a local maximum or a saddle point.

   b) (10 marks) Find the minimum value of the function $f(x,y,z) = \frac{1}{x} + \frac{2}{y} + \frac{2}{z}$, where $x > 0, y > 0, z > 0$, subject to the constraint $4x^2 + y^2 + 8z^2 = 1$.

5. a) (10 marks) Find the volume of the solid that lies under the surface $z = 1 + x + xy$ and above the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,2)$ in the *xy*-plane.

   b) (10 marks) Evaluate the iterated integral $\int_0^1 \int_{y^2}^1 \frac{8y^5}{1+x^4}\, dx\, dy$.