4) a) $\sqrt{(5-x)^2+(6-y)^2+(-4-z)^2} = 2\sqrt{(2-x)^2+(0-y)^2+(-1-z)^2}$

$(5-x)^2+(6-y)^2+(-4-z)^2 = 4[(2-x)^2+(-y)^2+(-1-z)^2]$

$25-10x+x^2+36-12y+y^2+16+8z+z^2 = 4[4-4x+x^2+y^2+1+2z+z^2]$

$x^2+y^2+z^2-10x-12y+8z+77 = 4x^2+4y^2+4z^2-16x+8z+20$

$57 = 3x^2-6x+3y^2+12y+3z^2$

$57 = 3(x^2-2x)+3(y^2+4y)+3z^2$

$57 = 3(x^2-2x+1-1)+3(y^2+4y+4-4)+3z^2$

$72 = 3(x-1)^2+3(y-2)^2+3z^2$

$24 = (x-1)^2+(y-2)^2+z^2$

$\boxed{\text{centre: } (1,2,0) \quad \text{radius: } \sqrt{24}}$

b) $r(t) = (t^2+2t, \sin t, e^{2t})$

$r'(t) = (2t+2, \cos t, 2e^{2t})$

$r'(0) = (2, 1, 2)$ $\quad |r'(0)| = \sqrt{4+1+4} = \sqrt{9} = 3$

$T(t) = \frac{r'(t)}{|r'(t)|}$ thus $\boxed{T(0) = \frac{(2,1,2)}{3}}$

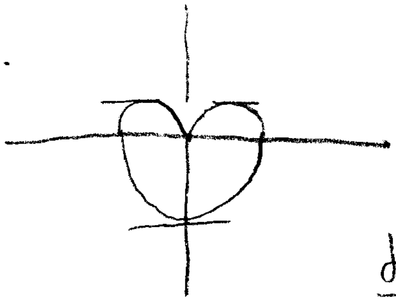
$k = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$ $\quad r''(t) = (2, -\sin t, 4e^{2t})$

k at $t=0$ so $k = \frac{|r'(0) \times r''(0)|}{|r'(0)|^3}$

$= \frac{\sqrt{36}}{3^3}$

$\boxed{= \frac{6}{27}}$

5) a) $r = 1 - \sin\theta \qquad 0 \le \theta \le 2\pi$



where $m = 0$

so $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

$\dfrac{dy}{dt} = (-\cos\theta)(\sin\theta) + (1 - \sin\theta)(\cos\theta)$

$= \cos\theta(-\sin\theta + 1 - \sin\theta) = 0$

$= \cos\theta(1 - 2\sin\theta) = 0$

$\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta - \sin 2\theta = 0$

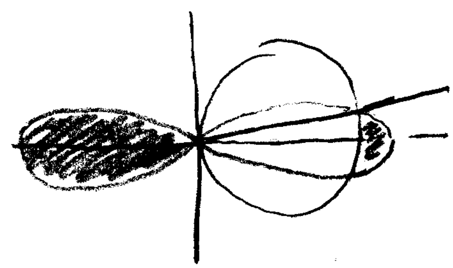
$\cos\theta = \sin 2\theta$

$\theta = \pi/6,\ 5\pi/6,\ 3\pi/2$

thus coordinates are

$(1/2, \pi/6),\ (\tfrac{1}{2}, 5\pi/6),\ (2, 3\pi/2)$

b) $r^2 = 6\cos(2\theta)$ and $r = 2\cos\theta$



curves intersect when $r^2 = r$

$6\cos(2\theta) = 4\cos^2\theta$

$3/2(1 - 2\sin^2\theta) = 1 - \sin^2\theta$

$2\sin^2\theta = \dfrac{1}{2}$

$\sin\theta = \pm\dfrac{1}{2}$

$\theta = \pi/6$

find area using symmetry

$A = 2\int_0^{\pi/4} \frac{1}{2}(6\cos 2\theta)\,d\theta = 6\int_0^{\pi/4} \cos 2\theta\,d\theta = 6\left[\frac{\sin 2\theta}{2}\Big|_0^{\pi/4}\right] = 6\left[0 + \frac{1}{2}\right] = 3$

$A_D = \int_0^{\pi/6} \frac{1}{2}(6\cos 2\theta)\,d\theta - \int_0^{\pi/6} \frac{1}{2}(2\cos\theta)^2\,d\theta$

$= 3\int_0^{\pi/6} \cos 2\theta\,d\theta - 2\int_0^{\pi/6} \cos^2\theta\,d\theta$

$= 3\left[\frac{\sin 2\theta}{2}\Big|_0^{\pi/6}\right] - 2\left[\frac{1}{2}\theta + \frac{\cos 2\theta}{4}\Big|_0^{\pi/6}\right]$

$= 3\left[\frac{1}{4}\right] - 2\left[\frac{\pi}{12} + \frac{\sqrt{3}}{8} - \frac{1}{4}\right]$

$= \frac{3}{4} - \frac{\pi}{6} - \frac{\sqrt{3}}{4} + \frac{2}{4}$

$= \frac{5}{\ } - \frac{\pi}{\ } - \frac{\sqrt{3}}{\ } \times 2 = \frac{5}{\ } - \frac{\pi}{\ } - \frac{\sqrt{3}}{\ }$

thus the area is

$\boxed{A = \frac{11}{2} - \frac{\pi}{3} - \frac{\sqrt{3}}{2}}$