**University of Toronto**
**Department of Mathematics**

**MAT 235Y1Y**
**Calculus II**

**TERM TEST # 2**
**Tuesday, January 23, 2001**

**Last Name:** ______________________

**Given Name:** ______________________

**Student Number:** ______________________

**INSTRUCTIONS:**

- Answer all questions in the space provided.
- No aids are allowed.

| FOR MARKERS ONLY | |
|---|---|
| Question | Mark |
| 1 | / 10 |
| 2 | / 20 |
| 3 | / 30 |
| 4 | / 15 |
| 5 | / 15 |
| 6 | / 10 |
| TOTAL | / 100 |

1. *[15 marks]*

**Find the equations of the tangent plane and normal line of the surface given by the equation:** $xyz - 4xz^3 + y^3 = 10$ **at the point** $(-1, 2, 1)$.

2. *[15 marks]*

Let $f(x,y) = \arctan(y/x)$, $P$ the point $(4,-4)$, and $\mathbf{X}$ the vector $2\mathbf{i} - 3\mathbf{j}$. Find the directional derivative of $f$ at $P$ in the direction of $\mathbf{X}$. Also find a unit vector in the direction of which the directional derivative is maximum. What is the value of the maximum directional derivative.

3. *[15 marks]*

Let $f(x,y) = x^2 - 4xy + y^3 + 4y$.

(i) Find the critical points of $f$.

(ii) Ascertain which of the critical points are points of local maximum, minimum or saddle point.

4. (a) *[10 marks]*

A function $f(x,y)$ is said to be *harmonic* if $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$. Prove that the following functions are *harmonic*:

(i) $f(x,y) = log(\sqrt{x^2+y^2})$

(ii) $f(x,y) = e^{-x}cos(y) + e^{-y}cos(x)$

4. (b) *[10 marks]*

If $z = f(x, y)$ where $x = rcos\theta$, $y = rsin\theta$, find $\dfrac{\partial^2 z}{\partial r \partial \theta}$ in terms of the first and second partial derivatives of $z$ with respect to $x$ and $y$ and functions of $r$ and $\theta$.

5. (a) *[10 marks]*

The law for an ideal gas may be stated as $PV = cnT$, where $P$ = Pressure, $V$ = Volume, $T$ = Temperature, $n$ = no. of moles in the gas and $c$ = a constant. Therefore, each of the variables $P, V, T$ can be regarded as a function of the other two. Show that

$$\frac{\partial V}{\partial T}\frac{\partial T}{\partial P}\frac{\partial P}{\partial V} = -1$$

5. (b) *[10 marks]*

Suppose $z$ is defined *implicitly* as a function of $(x, y)$ by the equation : $x^2y+y^2z+2xz^3 = 4$ .

Find $\dfrac{\partial z}{\partial x}$ , $\dfrac{\partial z}{\partial y}$ when $x = 1, y = 0, z > 0$ .

6. *[15 marks]*

Let $f(x, y, z) = 4x^2 + y^2 + 5z^2$ . Find the point on the plane : $2x + 3y + 4z = 12$ at which $f$ attains its extremum value.