PLEASE HAND IN

UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATIONS, APRIL 1999
MAT 235 Y – CALCULUS FOR PHYSICAL AND LIFE SCIENCE II

EXAMINERS: P. GREINER, F. RECIO, D. K. SEN, R. STANCZAK

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. NO CALCULATORS OR ANY OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF EIGHT QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 40% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 3 HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	10	
2	10	
3	15	
4	10	
5	15	
6	15	
7	10	
8	15	
TOTAL:	100	

NAME: ______________________ , ______________________

(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

1. a) (5 marks) Suppose that an object is moving through space along the path described by the position function $\mathbf{f}(t) = (t - 2, 1 - t^2, t + t^2)$, and then at the moment when $t = 1$ it suddenly begins travelling in the direction of its velocity vector $\mathbf{f}'(1)$ with $\|\mathbf{f}'(1)\|$ as its constant speed. What will be the object's position when $t = 3$?

   b) (5 marks) Let α be the plane that passes through the points $(2, 0, -1)$, $(1, -1, 0)$ and $(1, -2, -1)$. Find the coordinates of the point on the plane α which is closest to the point $(-4, 1, 0)$.

CONT'D

2\. a) (5 marks) Suppose that $\mathbf{T} : \mathrm{R}^2 \to \mathrm{R}^2$ is the linear transformation represented by the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}$. Find a vector $\mathbf{x}$, if any, for which $\mathbf{T}(\mathbf{T}(\mathbf{x})) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

b) (5 marks) Let $p(x, y, z) = k x^2 + y^2 + z^2 + 4xy + 2kxz + 2yz$. Find a value of k, if any, for which the given quadratic form $p(x, y, z)$ is positive definite.

3. a) (5 marks) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{1+3y^2-\cos x}{x^2+y^2}$ does not exist.

b) (5 marks) Let $f(x,y)=(x^5+8y^3)^{1/3}$. Compute $\dfrac{\partial f}{\partial y}(0,0)$.

c) (5 marks) Let $g(x,y)=xy^3/e^y$, where $x=2t^3-1+t^{-1}$ and $y=-t+3-3t^{-1}$.

Evaluate dg/dt at $t=1$.

4. (10 marks) A rectangular box is to be inscribed in the tetrahedron whose faces are the coordinate planes and the plane $4x + 3y + 6z = 12$. One corner of the box touches this plane, the opposite corner is at the origin, and the faces of the box are parallel to the coordinate planes. Find the volume of the largest such box.

5. a) (7 marks) Find the mass of a wire in the shape of the curve traced by $(3t, 3t^2, 4t^{3/2})$ where $0 \le t \le 1$, if its density at each point is equal to the distance between that point and the yz-plane.

b) (8 marks) Evaluate the surface integral $\iint\limits_M \frac{z}{\sqrt{1+8z-4y^2}}\, d\sigma$, where the surface M is the portion of the elliptic paraboloid $z = 2x^2 + y^2$ that lies above the rectangular region $R = [0, 1] \times [0, 2]$.

6. (15 marks) Evaluate the triple integral $\iiint\limits_{S} z^3 \, dx\, dy\, dz$, where S is the solid bounded above by the sphere $x^2+y^2+z^2=2$ and below by the cone $z=(3x^2+3y^2)^{1/2}$.

7. (10 marks) Evaluate the line integral $\oint_C (1+3x^2y+y^2)dx+(x^3+3xy+y)dy$, where the closed curve C is the boundary of the region enclosed between the parabolas $x+y^2=4$ and $x+2y-y^2=0$, oriented counterclockwise.

8. (15 marks) Let the solid S be the portion of the cylindrical region $x^2 + y^2 \leq 3$ that lies inside the ellipsoid $x^2 + y^2 + 4z^2 = 4$ and let ∂S denote the complete boundary of the solid S. Let $\mathbf{F}(x, y, z) = (x^3, x - z^2, 3y^2 z)$. Find the flux of the vector field $\mathbf{F}$ out of the closed surface ∂S. That is: evaluate the flux integral $\oint\!\!\oint_{\partial S} (\mathbf{F} \cdot \mathbf{n})\, d\sigma$.