FINAL EXAMINATIONS, APRIL 1995
MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCES II
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Each Question is worth 10marks each.

1. Let L_1 be the straight line through the points (5,2,-3) and (4,6,-6). Let L_2 be the straight line with symmetric equations $\frac{x}{3}=\frac{y+6}{-4}=\frac{z-2}{5}$.
   a) (3 marks) Find parametric equations for the line L_1.
   b) (3 marks) Find a unit vector **v** such that **v** is orthogonal to both lines L_1 and L_2.
   c) (3 marks) Find an equation of the plane that contains the line L_2 and is parallel to the line L_1.
   d) (3 marks) Find the coordinates of the point of intersection of the line L_2 and the plane $x+2y-z=6$.

2. The position vector $\mathbf{r}(t)=(2t\sin t, 2t\cos t, 1+\frac{t^3}{3})$ describes the motion of an object along a curve C.
   a) (3 marks) Show that $\|\mathbf{r}'(t)\|=2+t^2$, for all values of t.
   b) (3 marks) Find the angle between the velocity vector and the horizontal plane at $t=\sqrt{2}$.
   c) (2 marks) Find the arclength of the curve C from t=0 to t=1.

3. a) (5 marks) Given the function $f(x,y)=\frac{y}{x^2-y}$, where x and y are defined as implicit functions of the variables r and s by the equations $rx^2+2x-s+3=0$ *and* $3sy+r\sqrt{y}-2s^2-7=0$.
   Compute $\frac{\partial f}{\partial s}$ at r=0, s=1.
   b) (5 marks) Let S be the surface $z=xe^{\frac{y}{x}}-3x$. Show that for any point P=(a,b,c) on the surface S, the line joining the points (0,0,0) and P is tangent to the surface S.

4. Given the function $f(x,y)=xy+24\ln x-\frac{3}{2}y^2-18y$, where $x>0$.
   a) (3 marks) Find all critical points of this function.
   b) (3 marks) Use the Second Derivative Test to classify each of the critical points found in part (a) as a saddle point, a point where f(x,y) has a local maximum or a point where f(x,y) has a local minimum.
   c) (4 marks) Find the absolute maximum and the absolute minimum of the function f(x,y) over the triangle $1\le x\le 3$, $\frac{2}{3}x\le y\le 2$.

5. (10 marks) A plane passes through the point (1,2,3) and intersects the coordinate axes at the points (a,0,0), (0,b,0), and (0,0,c), with a>0, b>0, and c>0. Find the values of a, b, and c such that the region bounded by this plane and the three coordinate planes in the first octant has the smallest volume.

6. a) (5 marks) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{\sqrt{1+y^3}} \, dydx$.

b) (5 marks) Evaluate $\iiint_R \frac{1}{1+x^2} dV$, where R is the region enclosed in the first octant between the planes y=2, z=1, and z=x.

7. (10 marks) Compute $\iint_R (y^2-xy)dA$, where R is the parallelogram enclosed by the lines y-x=0, y-x=2, 3x-y=0, and 3x-y=1.

Hint: Use a change of variables.

8. (10 marks) Let $R=\{(x,y,z) : 0\le x,\ 0\le y,\ x+3y\le 6,\ 0\le z\le y^2\}$. Let S be the boundary of R, and let n(x,y,z) denote an outer unit normal vector at each point of the surface S. Compute $\iint_S (F\cdot n)dS$, where $F(x,y,z)=(y^2+x^2z,\ 2xyz,\ 1+yz-2xz^2)$.

9. (10 marks) Verify Stokes' Theorem when F(x,y,z)=(1,2xz,xy), and S is the surface $z=7-x^2-y^2$, $z\ge 3$.