DEPARTMENT OF MATHEMATICS
University of Toronto
MAT 235Y
Test #2
Wed, Jan 12, 1994
6:30 - 8:30 p.m.

7 Problems, 10 Marks each.

1. Sketch some level curves of the function

$$f(x,y) = x^2 - 2x + 2y^2 + 4y$$

on the region $-2 \leq x, y \leq 2$. Add a little commentary describing the most interesting features of the function on this region.

2. Consider the function $f(x,y) = x^3 + 6xy + 3y^2 + 12x + 12y$ defined on the entire x-y plane.

a. Find all the critical points of f.

b. For each critical point, determine whether it is a local minimum, a local maximum, or a saddle.

3. Find the absolute maximum and minimum of the function

$$f(x,y) = (x^2 + y^2)e^x$$

on the disk $x^2 + y^2 \leq 9$.

4. Find the absolute maximum and minimum of the function

$$f(x,y) = x^2 - xy$$

on the square region $-1 \leq x, y \leq 1$.

5. A storage shed is being built in the shape of a rectangular box. Material for the floor costs \$50 per square meter. Material for the walls costs \$20 per square meter. Material for the roof costs \$30 per square foot. What are the dimensions for a shed of the largest volume that can be constructed for \$3840 worth of material.

6. Find the point on the intersection of the planes $z = 2$ and $x - y + z = 3$ which is closest to the point $(2, 1, -1)$.

7. Find the least squares linear approximation $y = mx + b$ to the data points $(1,0)$, $(2,1)$, $(3,4)$. Here you are required to minimize the relevant function of m and b by taking its gradient and setting it equal to zero; you are not allowed to simply plug in to any general least-squares formula you may remember.