Quiz 2 Solutions

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Problem 1. (a) The ordinary generating function is

$$(x^0 + x^2 + x^4 + x^6 + \cdots)^8$$

(b) We know that $\frac{1}{1-x} = x^0 + x^1 + x^2 + x^3 + \cdots$. Now replacing x with x^2 in this, we get

$$\frac{1}{1-x^2} = x^0 + x^2 + x^4 + x^6 + \cdots$$

So we want the co-efficient of x^{20} in $\frac{1}{(1-x^2)^8}$. We know

$$\frac{1}{(1-x^2)^8} = \binom{0+8-1}{0}x^0 + \binom{1+8-1}{1}x^2 + \binom{2+8-1}{2}x^4 + \binom{3+8-1}{3}x^6 + \cdots$$

So the co-efficient of x^{20} is $\binom{10+8-1}{10}$

Problem 2. (a) The exponential generating function is

$$\left(\frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)^2 \left(\frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$

(b) The generating function is equal to $(e^x - 1)^2 e^x$. We want the co-efficient of x^r in this. We have

$$(e^x - 1)^2 e^x = (e^{2x} - 2e^x + 1)e^x = e^{3x} - 2e^{2x} + e^x.$$

Using the Taylor series for e^x this is equal to

$$\sum_{r=0}^{\infty} \frac{3^r x^r}{r!} - \sum_{r=0}^{\infty} \frac{2^{r+1} x^r}{r!} + \sum_{r=0}^{\infty} \frac{x^r}{r!}$$

So the co-efficient of $\frac{x^r}{r!}$ is $3^r - 2^{r+1} + 1$.