Quiz 2 Solutions

Dilip Raghavan

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Problem 1. (a) It is best to count the complement: the number of distributions where one of the 2 boxes is empty. First to count the total number of distributions, first distribute the red balls: $\binom{4+2-1}{4}$ ways. Next distribute the blue balls: $\binom{5+2-1}{5}$ ways. And finally the yellow balls: $\binom{7+2-1}{7}$ ways. So there are a total of $\binom{4+2-1}{4} \times \binom{5+2-1}{5} \times \binom{7+2-1}{7}$ ways to distribute the balls without restrictions. Now if one of the boxes is empty, then all the balls must go into the other box. So there are 2 distributions with one box empty. So the answer is $\binom{4+2-1}{4} \times \binom{5+2-1}{5} \times \binom{7+2-1}{7} - 2$

(b) Again count the complement. If the balls are all distinct, there are a total of 16 balls. The total number of distributions is then 2^{16} (for each ball you have 2 choices, box 1 or box 2). There are again 2 distributions where all the balls end up in one of the boxes. So the answer is $2^{16} - 2$.

Problem 2. For each integer between 1 and 15 we have 2 choices: we can put it in box 1 or in box 2. Since box 1 has exactly five integers we must use box 1 five times and so, we must use box 2 ten times. So we are really arranging five 1s and ten 2s in row. The requirement that consecutive integers cannot go into box 1 means that there are no consecutive 1s in the arrangement. So let us first arrange the five 1s:

$$1 \quad 1 \quad 1 \quad 1 \quad 1$$

Now we want to distribute the 2s making sure that there are no consecutive 1s. So first put a 2 into each of the 4 spaces between the 1s. Now, we are left with 6 2s to distribute into the 6 spaces. So there are $\binom{6+6-1}{6}$ ways to do this.