1. Let \( B(t) \) be Brownian motion. Prove that \( X = \int_0^1 B^2(s)ds \) is a random variable and compute the first two moments. Is \( X \) Gaussian?

2. Let \( B(t) \) be Brownian motion. Prove that with probability one,
\[
\lim_{n \to \infty} \sum_{i < 2^n t} |B(i+1)2^{-n} - B(i)2^{-n}|^2 = t
\]
(Hint: Compute the variance and use Borel-Cantelli)

3. Let \( p \in (-1/2, 1/2) \). For each fixed \( y \) prove that
\[
f_y(x) = |x - y|^{-p} - |x|^{-p}
\]
is in \( L^2(\mathbb{R}) \).
For a nice function \( f \) define \( X(f) = \int_{-\infty}^{\infty} f dB = -\int_{-\infty}^{\infty} f'(s)B(s)ds \).
Here \( B(t), t \in \mathbb{R} \) is a two sided Brownian motion, obtained as follows: Let \( B_1(t) \) and \( B_2(t) \) be independent Brownian motions starting at 0 and define \( B(t) = B_1(t) \) for \( t \geq 0 \) and \( B(t) = B_2(-t) \) for \( t < 0 \). Let
\[
Z_y = X(f_y)
\]
\( Z_y \) is called fractional Brownian motion of index \( \alpha \). Find the distribution (ie. all finite dimensional distributions) of \( Z_y \).

4. Let \( \tau \) be a stopping time. Show that
\[
\mathcal{F}_\tau = \{ A \in \mathcal{F} : A \cap \{ \tau \leq n \} \in \mathcal{F}_n, \ n \geq 0 \}
\]
is a \( \sigma \)-field.

5. If \( \tau \) and \( \sigma \) are stopping times, then so are \( \tau + \sigma \), \( \max(\tau, \sigma) \) and \( \min(\tau, \sigma) \).

6. Let \( B_t, t \geq 0 \) be Brownian motion. Use the law of the iterated logarithm
\[
\limsup_{t \to 0} \frac{B_t}{\sqrt{2t \log \log t}} = 1 \quad a.s.
\]
to compute all possible limit points of
\[
\frac{B_t}{\sqrt{\log \log t}}
\]
over subsequences \( t_n \to \infty \).