NONCONVENTIONAL LIMIT THEOREMS.

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ABSTRACT. The polynomial ergodic theorem (PET) which attracted substantial attention in ergodic theory studies the limits of expressions having the form $1/N\sum_{n=1}^N T^{q_1(n)}f_1\cdots T^{q_\ell(n)}f_\ell$ where T is a weakly mixing measure preserving transformation, f_i's are bounded measurable functions and q_i's are polynomials taking on integer values on the integers. Motivated partially by this result we obtain a central limit theorem for even more general expressions in both discrete time
$1/\sqrt{N}\sum_{n=1}^N \big(F\big(X_1(n), X_2(2n), ..., X_m(mn),$
$X_{m+1}(q_{m+1}(n)), X_{m+2}(q_{m+2}(n)), \cdots, X_\ell(q_\ell(n))\big) - \bar F\big)$
and in the continuous time for the corresponding expressions of the form
$1/\sqrt{T}\int_0^T \big(F\big(X_1(t), X_2(2t), ..., X_m(mt)X_{m+1}(q_{m+1}(t)), X_{m+2}(q_{m+2}(t)),$
$\cdots, X_\ell(q_\ell(t))\big) - \bar F\big)dt$
where X_i's are exponentially fast ψ-mixing bounded processes with some stationarity properties, F is a Lipschitz continuous function, $\bar F = \int F d(\mu_1\times\cdots\times\mu_\ell)$, μ_j is the distribution of $X_j(0)$ and $q_i, i > m$ are positive functions taking on integer values on integers in the discrete time case with some growth conditions which are satisfied, for instance, when q_i's are polynomials of growing degrees. When $F(x_1, ..., x_\ell) = x_1x_2\cdots x_\ell$ exponentially fast α-mixing already suffices. This result can be applied in the case when $X_i(n) = T^n f_i$ where T is a mixing subshift of finite type, a hyperbolic diffeomorphism or an expanding transformation taken with a Gibbs invariant measure, as well, as in the case when $X_i(n) = f_i(\xi_n)$ where ξ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure. In the continuous time case the result can be applied to $X_i(t) = f_i(\xi_t)$ where ξ_t is a nondegenerate diffusion on a compact manifold. Also some large deviations results in this setup were obtained recently together with Varadhan.