The mean perimeter of some random plane convex sets generated by a Brownian motion*

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Abstract

If C_1 is the convex hull of the curve of the standard Brownian motion in the complex plane watched from 0 to 1, and if w a primitive n th root of unity, we consider the convex hull C_n of $C_1 \cup wC_1 \ldots \cup w^{n-1}C_1$. For instance C_2 is the symmetrized convex hull of the Brownian curve. We compute the means of the perimeters of C_1, C_2, C_4 by elementary calculations as well as some other simple convex hulls. The computation of the means of the perimeter of C_3 and C_6 is more involved and is done by the computation of the distribution of the exit time by the standard Brownian motion of the fundamental domain for symmetry groups in Euclidean spaces (joint work with Philippe Biane).

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