# MAT135Y – 2007-2008

## Review Problems for Term-Test 3

The following problems were given at third term-tests of previous academic years. For each problem there is a year followed by a number. The year is the year at which the problem was given and the number is the number of the problem in the Term-Test 3 booklet of that year. For instance ['04, A.6] refers to Problem A.6 of Term-Test 3 of March 2004. You can get the answers of the problem by looking at the solutions of the corresponding term-test. These solutions are available online at:

`http://www.math.utoronto.ca/ponge/teaching/2007-08/MAT135/MAT135.html`.

The sign $\star$ indicates that the problem is chalenging.

### Problems on Section 4.10 and Chapter 5

**Problem 1** ('06, A.1)**.** If $F(x)$ is the antiderivative of $f(x) = 3x^2 - 2x + 5$ such that $F(1) = 0$, then $F(2) =$

(A) 12 (B) 0 (C) 7 (D) 9 (E) $-2$.

**Problem 2** ('06, A.2)**.** If $\int_0^2 \{4f(x) + 3g(x)\}\, dx = 2$ and $\int_0^2 f(x)dx = 5$, then $\int_0^2 g(x)dx =$.

(A) $-6$ (B) 0 (C) 8 (D) $-5$ (E) 7.

**Problem 3** ('03, A.4)**.** If $F(x) = \int_2^{3x} \sqrt{t^3 - 2}dt$, then $F'(1) =$?

(A) 14 (B) undefined (C) 13 (D) 12 (E) 15.

**Problem$^\star$ 4** ('06, A.12)**.** Let $G(x) = \int_{3x}^{2x} f(t)dt$, where $f(t) = \int_{2t-5}^{2-t} \sqrt{1 + u^4}du$. Then $G''(1) =$

(A) $-4 + 19\sqrt{2}$ (B) $3 + 20\sqrt{2}$ (C) $1 - 17\sqrt{2}$ (D) $-2 - 18\sqrt{2}$ (E) $5 + 16\sqrt{2}$.

**Problem$^\star$ 5** ('04, A.12)**.** Find the value of $\lim_{n\to\infty} \left( \frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + \ldots + \frac{n^2}{n^3+n^3} \right)$.

(1) $\frac{1}{2}\ln 2$ (2) $\frac{1}{2}\ln 3$ (3) $\frac{1}{3}\ln 2$ (4) undefined (5) $\frac{1}{3}\ln 3$.

**Problem$^\star$ 6** ('06, B.7)**.** Given that $\int_0^4 e^{(x-2)^4}dx = k$, find the value of $\int xe^{(x-4)^4}dx$ in terms of the constant $k$.

### Problems on Chapter 6

**Problem 7** ('03, A.6)**.** Find the area enclosed between the curves $y = 4x^2$ and $y = 8x$.

(A) $\frac{12}{5}$ (B) $\frac{14}{5}$ (C) $\frac{16}{3}$ (D) $\frac{13}{2}$ (E) $\frac{15}{2}$.

**Problem 8** ('06, A.5)**.** Find the area enclosed between the curves $y = \frac{1}{2}x^4$ and $y = 4x$.

(A) $\frac{28}{5}$ (B) $\frac{26}{5}$ (C) $\frac{24}{5}$ (D) $\frac{32}{5}$ (E) $\frac{18}{5}$.

**Problem 9** ('06, A.6)**.** Let $R$ be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = x$. Find the volume generated by revolving the region $R$ about the $x$-axis.

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{4}$.

**Problem 10** ('04, A.7)**.** Let $R$ be the region enclosed between the curves $y = x$ and $y = \sqrt{x}$. Find the volume of the solid generated by resolving the region $R$ about the $y$-axis.

(1) $\frac{2\pi}{15}$ (2) $\frac{\pi}{7}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{8}$ (5) $\frac{3\pi}{14}$.

**Problem 11** ('05, A.7)**.** The average value of $f(x) = 3 + 2x + 4x^3$ on $[0, 2]$ is

(A) 13 (B) 18 (C) 8 (D) 10 (E) 15.

**Problem$^\star$ 12** ('03, A.9)**.** Find the area of the region enclosed between the curve $x = y^2$ and the right semi-circle $x^2 + y^2 = 2$, $x \geq 0$.

(A) $\frac{2}{3} + \frac{\pi}{4}$ (B) $\frac{4}{3} + \frac{\pi}{4}$ (C) $\frac{3}{4} + \frac{\pi}{4}$ (D) $\frac{1}{3} + \frac{\pi}{2}$ (E) $\frac{4}{3} + \frac{\pi}{2}$.

**Problem$^\star$ 13** ('04, A.11)**.** Find the area of the region enclosed between the curves $x - y + y^2 = 0$ and $x + x^2 + 2y = 0$. Hint: The two curves intersect at the points $(0, 0)$ and $(-2, -1)$.

(1) $\frac{5}{6}$ (2) $\frac{6}{5}$ (3) $\frac{7}{2}$ (4) $\frac{3}{4}$ (5) $\frac{4}{5}$.

**Problem$^\star$ 14** ('05, B.6)**.** Let $k$ be a number between 0 and $\pi$. Let $R$ be the region in the first quadrant bounded by the $x$-axis, the line $x = k$ and the curve $y = \sin x$, $0 \leq x \leq k$. If the volume generated by revolving the region $R$ about the $x$-axis is $\frac{\pi}{24}(5\pi - 3)$, then find the value of $k$.

**Problem$^\star$ 15** ('06, A.9)**.** Let $R$ be the region the bounded by the $x$-axis, the line $x = 1$ and the line $y = 2x$. Let $V$ be the volume of the solid generated by revolving the region $R$ about the line $x = a$ (where $a > 1$). Then $a =$

(A) $\frac{2\pi + 3V}{3\pi}$ (B) $\frac{2\pi + V}{3\pi}$ (C) $\frac{4\pi + 5V}{6\pi}$ (D) $\frac{2\pi + 2V}{3\pi}$ (E) $\frac{4\pi + 3V}{6\pi}$.

## Problems on Chapter 7

**Problem 16** ('06, B.1)**.** Find $\int x \sin x dx$.

**Problem 17** ('06, B.2)**.** Find $\int \frac{1}{x(2+\ln x)^3} dx$.

**Problem 18** ('05, B.1)**.** Find $\int xe^{4x} dx$.

**Problem 19** ('05, B.5)**.** Find $\int \frac{2x^2+5x-4}{x(x+2)(x-1)} dx$.

**Problem 20** ('06, B.5)**.** Find $\int \frac{5x^2+4x+2}{x(x^2+1)} dx$.

**Problem 21** ('03, B.2)**.** Find $\int \sin^{62} x \cos^3 x dx$.

**Problem 22** ('06, B.3)**.** Find $\int \tan^{78} x \sec^4 x dx$.

**Problem 23** ('04, B.2)**.** Find $\int x \sec^2(x^2) \tan^4(x^2) dx$.

**Problem 24** ('03, B.3)**.** Find $\int \frac{1}{x^2\sqrt{25-x^2}} dx$.

**Problem 25** ('06, B.4)**.** Find $\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx$.

**Problem 26** ('06, B.6)**.** Find $\int \frac{dx}{1+\sqrt{x}} dx$.

**Problem 27** ('04, A.9)**.** $\int_0^\infty (e^{-2x} - e^{-3x}) dx =$

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{6}$ (4) $\infty$ (5) $\frac{1}{3}$.

**Problem 28** ('06, A.8)**.** Find the value of $\int_1^\infty x^4 e^{-x^5} dx$.

(A) $+\infty$ (B) $\frac{1}{e}$ (C) $1+\frac{e}{5}$ (D) $\frac{e}{5}$ (E) $\frac{1}{5e}$.

**Problem 29** ('03, A.8)**.** Find the value of the constant $c$ such that $\int_0^c xe^{-x^2} dx = \int_c^\infty xe^{-x^2} dx$.

(A) $\ln 2$ (B) $\sqrt{2\ln 2}$ (C) $2\ln 2$ (D) $\frac{1}{2}\ln 2$ (E) $\sqrt{\ln 2}$.

**Problem$^\star$ 30** ('06, A.10)**.** $\int_{\sqrt{2}-1}^1 \frac{1}{\sqrt{x^2+2x}} dx =$.

(A) $\ln\left(\frac{1+\sqrt{3}}{1+\sqrt{2}}\right)$ (B) $\ln\left(\frac{1+\sqrt{3}}{2+\sqrt{2}}\right)$ (C) $\ln\left(\frac{2+\sqrt{3}}{1+\sqrt{2}}\right)$ (D) $\ln\left(\frac{2+\sqrt{3}}{3+\sqrt{2}}\right)$ (E) $\ln\left(\frac{2+\sqrt{3}}{2+\sqrt{2}}\right)$.

**Problem$^\star$ 31** ('04, A.10)**.** $\int_0^1 \frac{xe^x}{1+2x+x^2} dx =$

(1) $\frac{1}{2}(e-1)$ (2) $\frac{1}{2}(e-2)$ (3) $\frac{1}{4}(e-2)$ (4) $2(e-1)$ (5) $\frac{1}{4}(e-1)$.

**Problem$^\star$ 32** ('03, A.10)**.** $\int_0^{\frac{\pi}{4}} \frac{dx}{1+2\cos^2 x} dx =$.

(A) $\frac{\pi}{6\sqrt{3}}$ (B) $\frac{\pi}{4\sqrt{2}}$ (C) $\frac{\pi}{3\sqrt{3}}$ (D) $\frac{\pi}{6\sqrt{2}}$ (E) $\frac{\pi}{3\sqrt{2}}$.

**Problem$^\star$ 33** ('05, B.7)**.** Find $\int \frac{dx}{1+4\sin^2 x}$.

**Problem$^\star$ 34** ('04, B.7)**.** If $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^3(x) dx = L$, then find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^5(x) dx$ in terms of $L$.