MAT135Y – 2007-2008

Review Problems for Term-Test 2

The following problems were given at second term-tests of previous academic years. For each problem there is a year followed by a number. The year is the year at which the problem was given and the number is the number of the problem in the Term-Test 2 booklet of that year. For instance ['03, 6] refers to Problem 6 of Term-Test 2 of December 2003. You can get the answers of the problem by looking at the solutions of the corresponding term-test. These solutions are available online at:

`http://www.math.utoronto.ca/ponge/teaching/2007-08/MAT135/MAT135.html`.

The sign $\star$ indicates that the problem is chalenging.

Problem on Chapter 2

Problem 1 ('03, 6). Let

$$f(x) = \begin{cases} \frac{\sin kx}{\sin 2x} & \text{if } x < 0, \\ (x+k)^2 + (5k+2)(x+\frac{1}{2}) & \text{if } x \geq 0. \end{cases}$$

Find the value of the constant k so that f is continuous everywhere.

(a) 0 (b) 2 (c) -1 (d) $\dfrac{5}{2}$ (e) $\dfrac{1}{2}$.

Problems on Chapter 3

Problem 2 ('03, 7). The line tangent to the curve $x^2 + 3y^2 = 1$ at the point $(\frac{1}{2}, \frac{1}{2})$ intercept the y-axis at the point

(a) $(0, \dfrac{1}{3})$ (b) $(0, \dfrac{1}{2})$ (c) $(0, \dfrac{4}{3})$ (d) $(0, 1)$ (e) $(0, \dfrac{2}{3})$.

Problem* 3 ('02, 18). Let $f^{(n)}(a)$ denote the n'th derivative of f at a. If $f(x) = e^{2x}\cosh(2x)\sinh(4x)$, then $f^{(20)}(\ln 4) = ?$.

(a) $2^{74} + 2^{46} - 2^{30}$ (b) $2^{72} + 2^{46} - 2^{32}$ (c) $2^{72} + 2^{44} - 2^{32}$ (d) $2^{72} + 2^{46} - 2^{30}$ (e) $2^{74} + 2^{44} - 2^{30}$.

Problem 4 ('04, 7). Let $f^{(n)}(a)$ denote the n'th derivative of f at a. If $f(x) = \sin 2x$, then $f^{(6)}(\frac{\pi}{12}) =$.

(a) 16 (b) -32 (c) 64 (d) 0 (e) -16.

Problem 5 ('02, 5). If $xy^3 + y - x = 23$, then find the value of $\frac{dy}{dx}$ at the point where $x = 3$ and $y = 2$.

(a) $\dfrac{4}{25}$ (b) $\dfrac{5}{38}$ (c) $\dfrac{5}{37}$ (d) $-\dfrac{6}{35}$ (e) $-\dfrac{7}{37}$.

Problem 6 ('04, 6). If $y^2 + xy = 5$, then find the value of $\frac{dy}{dx}$ at the point where $x = 4$ and $y = -5$.

(a) $\dfrac{6}{5}$ (b) $-\dfrac{15}{4}$ (c) $\dfrac{15}{4}$ (d) $-\dfrac{5}{6}$ (e) $\dfrac{5}{2}$.

Problem 7 (‘03, 12). If $f(x) = x^{2x}$, then $f'(e) =$?

(a) e^{2e} (b) $2e^{2e}$ (c) $4e^2$ (d) e^2 (e) $4e^{2e}$.

Problem 8 (‘04, 8). If $f(x) = (\ln x)^x$, then $f'(e) =$?

(a) 0 (b) 2 (c) 1 (d) e (e) $\ln 2$.

Problem 9 (‘04, 5). If $f(x) = \arctan x$, then $f'(\sqrt{3}) =$

(a) $\dfrac{\sqrt{3}}{4}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $-\dfrac{\sqrt{3}}{8}$ (d) $-\dfrac{\sqrt{3}}{3}$ (e) $\dfrac{\sqrt{3}}{3}$.

Problem 10 (‘03, 4). If $f(x) = \arctan \frac{x}{2}$, then $f''(2) =$?

(a) -1 (b) 4 (c) 0 (d) $-\dfrac{1}{8}$ (e) $\dfrac{\pi}{4}$.

Problem 11 (‘04, 10). If $f(x) = \sinh x$, then $f''(\ln 3) =$?

(a) $\dfrac{1}{3}$ (b) $-\dfrac{1}{3}$ (c) e^3 (d) $\ln 3$ (e) $\dfrac{4}{3}$.

Problem 12 (‘03, 14). The length of a rectangle is *increasing* at the rate 5 cm/min, while its width is *decreasing* at the rate of 4 cm/min. At what rate will the area of the rectangle be changing when its length is 60 cm and its width is 40 cm?

(a) Increasing at 50 cm^2/min (b) Decreasing at 40 cm^2/min (c) Increasing at 60 cm^2/min

(d) Decreasing at 60 cm^2/min (e) Increasing at 40 cm^2/min.

Problem* 13 (‘04, 17). A spotlight on the ground shines on a wall 1,000 cm away. A boy walks from the spotlight toward the wall at the speed of 120 cm/sec. At the monent when the boy is 400 cm from the wall, the length of his shadow on the wall is decreasing at 46 cm/sec. How tall is the boy?

(a) 136 cm (b) 144 cm (c) 138 cm (d) 142 cm (e) 140 cm.

Problem* 14 (‘02, 20). A man 6 feet tall walks at 5 feet per second along one edge of a road that is 30 feet wide. On the other side of the road us a light atop a pole 18 feet high. How fast is the length of the man's shadow (on the horizontal ground) increasing when he is 40 feet beyond the point directly across the road from the pole?

(a) $\dfrac{5}{3}$ ft/sec (b) $\dfrac{7}{4}$ ft/sec (c) $\dfrac{3}{2}$ ft/sec (d) 2 ft/sec (e) $\dfrac{5}{4}$ ft/sec.

Problem 15 ('03, 11)**.** Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - x + 1$ on $[0, 2]$.

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{3}$ (e) $\frac{1}{3}$.

Problem 16 ('02,7)**.** Let $f(x) = x^2 - x$ on $[-2, 2]$. Let M be the absolute maximum of f on $[-2, 2]$ and let m be its absolute minimum. Then $M - m =$?

(a) $\frac{25}{4}$ (b) 4 (c) $\frac{11}{2}$ (d) $\frac{27}{4}$ (e) 6.

Problem 17 ('04, 12)**.** Let $f(x) = 3x^2 - 8x + 2$ on $[0, 3]$. Let M be the absolute maximum of f on $[0, 3]$ and let m be its absolute minimum. Then $M - m =$?

(a) 8 (b) 7 (c) $\frac{20}{3}$ (d) $\frac{25}{3}$ (e) $\frac{22}{3}$.

Problem 18 ('02, 8)**.** Let $f(x) = \frac{2}{5}x^5 - \frac{1}{2}x^4 - \frac{4}{3}x^3 + \frac{3}{2}$. At which point does f have a local maximum?

(a) -2 (b) 1 (c) 0 (d) 2 (e) -1.

Problem 19 ('04, 11)**.** Let $f(x) = x^4 + 8x^3 + 16x^2$. Then f has a local maximum at $x =$

(a) -2 (b) 0 (c) -4 (d) -8 (e) -6.

Problem 20 ('02, 10)**.** Let $f(x) = \frac{x^4}{12} - \frac{x^3}{6} - x^2 + 2x - 4$. On which intervals is the graph of f concave down?

(a) $(-1, 2)$ (b) $(-\infty, 1) \cup (3, \infty)$ (c) $(-\infty, -1) \cup (2, \infty)$ (d) $(1, 3)$ (e) $(-2, 1)$.

Problem 21 ('03, 9)**.** On what interval is the graph of $f(x) = (1 - \frac{1}{x})^2$ concave down?

(a) $(\frac{3}{2}, \infty)$ (b) $(1, \infty)$ (c) $(-1, 0)$ (d) $(0, 1)$ (e) $(-\infty, -1)$.

Problem 22 ('04, 13)**.** The graph of $y = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 3x^2 + x - 2$ is concave down on which interval

(a) $(-2, 3)$ (b) $(0, \infty)$ (c) $(-3, 2)$ (d) $(-1, 4)$ (e) $(-4, 1)$.

Problem 23 ('02, 9)**.** Find the y-coordinate of the inflection point of $f(x) = x^3 - x^2$.

(a) $\frac{2}{9}$ (b) $-\frac{2}{27}$ (c) $-\frac{2}{9}$ (d) $-\frac{2}{3}$ (e) $\frac{1}{3}$.

Problem* **24** ('04, 16)**.** If f is a function such that $f''(x) = (x - 1)^3(x + 2)(x^2 + 3x + 2)(x^{64} - 1)$, then how many inflection points does f have?

(a) none (b) one (c) three (d) two (e) more than three.

Problem 25 ('02, 1)**.** Find the value of $\lim_{x\to 0} \frac{x+\sin x}{4x^3-x}$.

(a) 0 (b) undefined (c) -1 (d) -2 (e) $\dfrac{1}{4}$.

Problem 26 ('02, 2)**.** Find the value of $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$.

(a) 0 (b) $+\infty$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$ (e) $-\dfrac{1}{4}$.

Problem 27 ('04, 15)**.** Find the value of $\lim_{x\to 0}(1-2x)^{\frac{1}{x}}$.

(a) $+\infty$ (b) 1 (c) $\dfrac{1}{2}$ (d) $\dfrac{2}{e}$ (e) e^{-2}.

Problem* 28 ('02, 17)**.** Find the value of $\lim_{x\to 0}\left(\frac{1}{x^2} - \cot^2 x\right)$.

(a) $\dfrac{3}{2}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{2}{3}$ (d) $-\dfrac{2}{3}$ (e) $-\dfrac{3}{2}$.

Problem* 29 ('03, 16)**.** Find the value of $\lim_{x\to 0} \frac{\sinh x - \sin x}{\sin^3 x}$.

(a) $-\dfrac{2}{3}$ (b) $\dfrac{1}{3}$ (c) undefined (d) $\dfrac{1}{6}$ (e) 0.

Problem 30 ('02, 14)**.** The slant asymptote of the curve $y = \frac{6x^3+9x^2+4x+5}{3x^2+6x}$ is the line

(a) $y = 2x-1$ (b) $y = 2x+5$ (c) $y = 2x+1$ (d) $y = 2x$ (e) $y = 2x + \dfrac{3}{2}$.

Problem 31 ('03, 10)**.** The graph of $y = \frac{5x^2+13x-7}{x+3}$ has one vertical asymptote and one other asymptote. The other asymptote is the line

(a) $y = 5x$ (b) $y = 5x-2$ (c) $y = 5x+3$ (d) $y = 5x+1$ (e) $y = 5x-3$.

Problem 32 ('02, 11)**.** The product of two positive numbers is 16. What is the smallest possible value of their sum?

(a) 6 (b) $\dfrac{15}{2}$ (c) 8 (d) 7 (e) $\dfrac{17}{2}$.

Problem 33 ('03, 13)**.** The product of two positive numbers is 9. What is the smallest possible value of the sum of their squares?

(a) 39 (b) $\dfrac{81}{2}$ (c) $\dfrac{163}{4}$ (d) 41 (e) 40.

Problem* 34 ('03, 19)**.** P is a point on the positive x-axis, Q is a point on the positive y-axis and O is the origin. What is the smallest possible area of the triangle OPQ if the line through P and Q is to be tangent to the curve $y = 4 - x^2$ at some point?

(a) $\dfrac{15\sqrt{3}}{4}$ (b) $4\sqrt{3}$ (c) $\dfrac{33\sqrt{3}}{10}$ (d) $\dfrac{32\sqrt{3}}{9}$ (e) $\dfrac{10\sqrt{3}}{3}$.

Problem* 35 ('04, 18)**.** P is a point on the positive x-axis, Q is a point on the positive y-axis. What is the shortest distance between P and Q if the line joining P and Q is to be tangent to the curve $y = \frac{2}{x}$ at some point?

(a) 3 (b) 4 (c) $\dfrac{7}{2}$ (d) $\dfrac{15}{4}$ (e) $\dfrac{9}{2}$.