

# MAT135Y – 2007-2008

## Review Problems for Term-Test 1

- The following problems were given at first term-tests of previous academic years. For each problem there is a year followed by a number. The year is the year at which the problem was given and the number is the number of the problem in the Term-Test 1 booklet of that year. For instance ['03, A.3] refers to Problem A.3 of Term-Test 1 of November 2003. You can get the answers of the problem by looking at the solutions of the corresponding term-test which are available online at:

<http://www.math.utoronto.ca/ponge/teaching/2007-08/MAT135/MAT135.html>.

- The sign  $\star$  indicates that the problem is challenging.
- If you would like to go over some problem during the review on Thursday, November 1st feel free to send me an e-mail about that.

### PROBLEMS ON CHAPTER 1

**Problem 1** ('03, A.3). If  $f(x) = \frac{3x+1}{2x-5}$ , then  $f^{-1}(x) = ?$

(a)  $\frac{5x+3}{2x-1}$       (b)  $\frac{5x+1}{2x-3}$       (c)  $\frac{5x-1}{2x+3}$       (d)  $\frac{5x-2}{2x+1}$       (e)  $\frac{5x-3}{2x+1}$ .

**Problem 2** ('04, A.3). If  $f(x) = \frac{4x-1}{2x+3}$ , then  $f^{-1}(x) = ?$ .

(a)  $\frac{4+3x}{1-2x}$       (b)  $\frac{1-4x}{3+2x}$       (c)  $\frac{1-4x}{3-2x}$       (d)  $\frac{4-2x}{1+3x}$       (e)  $\frac{1+3x}{4-2x}$ .

### PROBLEMS ON CHAPTER 2

**Problem 3** ('04, A.1). Find the value of  $\lim_{x \rightarrow 2} \frac{x^2-2x}{x^3+x^2-6x}$ .

(a) 0      (b) undefined      (c)  $\frac{1}{5}$       (d)  $-\frac{1}{3}$       (e)  $\frac{1}{2}$ .

**Problem 4** ('04, A.4). Find the value of  $\lim_{x \rightarrow -\infty} \frac{\sqrt{7x^6+1}}{3x^3-9}$ .

(a)  $-\frac{\sqrt{7}}{9}$       (b)  $-\frac{\sqrt{7}}{3}$       (c) undefined      (d)  $\frac{\sqrt{7}}{3}$       (e)  $\frac{\sqrt{7}}{9}$ .

**Problem 5** ('02, B.5). Let

$$f(x) = \begin{cases} 2x + c & \text{if } x < 3, \\ cx + c + 2 & \text{if } x \geq 3. \end{cases}$$

Find the value of  $c$  so that  $f$  is continuous everywhere.

**Problem 6** ('04, B.5(b)). Let

$$f(x) = \begin{cases} \frac{c(x^2-x-2)}{x-2} & \text{if } x > 2, \\ cx^2 + 1 & \text{if } x \leq 2. \end{cases}$$

Find a value of  $c$  such that  $f$  is continuous at 2. If such a value does not exist, explain why.

**Problem 7** ('02\*, B.6). Find the values of the constants  $a$  and  $b$  such that

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{-a|x| + b} - \sqrt{3}}{\sqrt{x + 4} - 2} = 4.$$

**Problem 8** ('04, A.5). The horizontal asymptote of the curve  $y = \frac{5x^3 - 2\sqrt{x+1}}{3+x^2-4x^3}$  is the line

$$(a) y = -\frac{5}{4} \quad (b) y = \frac{2}{3} \quad (c) y = -\frac{1}{4} \quad (d) y = \frac{1}{3} \quad (e) y = \frac{5}{3}.$$

**Problem 9** ('02, '03, '04, '05, B.1). Find  $f'(x)$  from first principles (i.e. by using only the definition of the derivative) for the following functions:

$$(a) f(x) = \sqrt{x}, \quad (b) f(x) = x^2, \quad (c) f(x) = \frac{1}{x}, \quad (d) f(x) = \sin x.$$

**Problem 10** ('04, B.5(a)). Let

$$f(x) = \begin{cases} x + 1 & \text{if } x \geq 0, \\ x + 2 & \text{if } x < 0. \end{cases}$$

Does  $f'(0)$  exist? Justify your answer.

**Problem 11** ('02, B.4). Find the line passing through the point  $(-3, 0)$  and tangent to the curve  $y = \sqrt{x-1}$  at some point. Note: the point  $(-3, 0)$  is not on the given curve.

**Problem 12** ('04, B.3). Find the line which passes through the point  $(0, -32)$  and is tangent to the curve  $y = 2x^3$  at some point.

**Problem 13** ('03\*, A.10). Let

$$f(x) = \begin{cases} -x^2 & \text{if } x \leq 0, \\ 8\sqrt{x} & \text{if } x > 0. \end{cases}$$

There is one line that is tangent to the graph of  $f$  at two points. What is its slope?

$$(a) 6 \quad (b) 2 \quad (c) 8 \quad (d) \sqrt{8} \quad (e) 4.$$

**Problem 14** ('02\*, A.10). There exists one, and only one, straight line that is tangent to the curve  $y = x^4 - 9x^2$  at two points. Find the distance between these two points.

$$(a) 2\sqrt{2} - 2 \quad (b) 3\sqrt{2} \quad (c) 4\sqrt{2} - 2 \quad (d) \sqrt{2} \quad (e) 2\sqrt{2}.$$

### PROBLEMS ON CHAPTER 3

**Problem 15** ('03, A.6). Evaluate  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$ .

$$(a) \frac{1}{4} \quad (b) -\frac{1}{2} \quad (c) 0 \quad (d) \text{undefined} \quad (e) \frac{1}{2}.$$

**Problem 16** ('04\*, B.6). Find the value of  $\lim_{x \rightarrow 1} \frac{x^{1/7}-1}{x^{1/5}-1}$ .

**Problem 17** ('03\*, B.6). Suppose that  $f$  is a differentiable function such that  $f'(8) = 2$ . Find the value of  $\lim_{x \rightarrow 8} \frac{f(x)-f(8)}{x^{1/3}-2}$ .

**Problem 18** ('04, A.6). If  $f(x) = \sin x \cos x$ , then  $f''(\frac{\pi}{4}) = ?$

- (a)  $-1$                       (b)  $-2$                       (c)  $\frac{1}{2}$                       (d)  $\pi$                       (e)  $\sqrt{2}$ .

**Problem 19** ('03, A.5). Find the value of  $\lim_{x \rightarrow 0} \frac{\sin(2x) \cos(3x)}{\sin(5x) \cos(4x)}$ .

- (a)  $\frac{3}{4}$                       (b)  $\frac{3}{5}$                       (c)  $0$                       (d) undefined                      (e)  $\frac{2}{5}$ .

**Problem 20** ('02, A.3). Find the value of  $\lim_{x \rightarrow 0} \frac{\sin(2x) \cos(3x) \sin(4x)}{x \cos(5x) \sin(6x)}$ .

- (a) undefined                      (b)  $\frac{4}{3}$                       (c)  $\frac{2}{5}$                       (d)  $\frac{1}{2}$                       (e)  $\frac{3}{5}$ .

**Problem 21** ('02\*, A.8). Find the value of  $\lim_{x \rightarrow 0} (\cot x - \csc x)$ .

- (a)  $0$                       (b) undefined                      (c)  $\frac{1}{2}$                       (d)  $1$                       (e)  $-1$ .

**Problem 22** ('02, B.2). Find any suitable method to find  $\frac{dy}{dx}$  for each of the following.

- a)  $y = (4 + x^2) \arcsin x$ ,  $-1 < x < 1$ ,      b)  $y = \frac{\sqrt{x}}{5 + e^x}$ ,      c)  $y = \arctan x^2$ ,      d)  $y = 2^{\sin 5x}$ .

**Problem 23** ('03, A.7). The graph of  $y = e^{3x^3 - 3x^2 + x - 4}$  has a horizontal tangent line at  $x =$

- (a)  $-\frac{3}{4}$                       (b)  $\frac{1}{3}$                       (c)  $\frac{3}{4}$                       (d)  $-3$                       (e)  $-\frac{1}{3}$ .

**Problem 24** ('03\*, B.5). Find  $\frac{dy}{dx}$  if  $y^3 + xy - 3y^2 - 2x - 2 = 0$ . What is the value of  $\frac{dy}{dx}$  when  $x = 2$ .

**Problem 25** ('02\*, A.9). If  $x^2 + xy + y^3 = 1$  find the value of  $\frac{d^3y}{dx^3}$  at the point where  $x = 1$ .

- (a)  $36$                       (b)  $44$                       (c)  $40$                       (d)  $38$                       (e)  $42$ .

**Problem 26** ('04\*, A.10). If  $y = \arctan(\sqrt{1+x^2} - x)$ , then  $\frac{dy}{dx} =$

- (a)  $\frac{2}{\sqrt{2+x^2}}$       (b)  $\frac{1+x}{\sqrt{1+x^2}-x}$       (c)  $\frac{1}{2(1+x^2)}$       (d)  $\frac{-1}{2\sqrt{1+x^2}}$       (e)  $\frac{-1}{2(1+x^2)}$ .

**Problem 27** ('03, A.8). A spaceship approaching touchdown on the planet GRY has height  $y = 100 - 100t + 25t^2$  meters at  $t$  seconds. What is its speed in meters per second when it hits the ground?

- (a)  $\frac{1}{2}$                       (b)  $1$                       (c)  $0$                       (d)  $\frac{3}{2}$                       (e)  $2$ .

**Problem 28** ('02, B.3). An object is thrown upward so its height above ground after  $t$  seconds is  $50 + 96t - 16t^2$  feet. Find the velocity of the object when it is 178 feet above ground on its way up.