Jordan Form

(1) Find the Jordan for the following operators.

(a) $T : V \to V$ defined by $T(f) = f'$ where V is the vectorspace of real valued functions spanned by $\{1, t, t^2, e^t, te^t\}$.

(b) $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by

$$T(A) = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} A - A^t$$

(c) Let $P_2[x, y]$ be the space of polynomials with coefficients from a field F of degree at most 2 (so, it is a 6-dimensional space with a basis $\{1, x, y, x^2, xy, y^2\}$). Let $T : P_2[x, y] \to P_2[x, y]$ be defined by

$$T(p) = \frac{\partial}{\partial x} p + \frac{\partial}{\partial y} p$$

(2) Let T be a linear operator on a finite dimensional vectorspace over a perfect field (just consider $\mathbb{C}$). Prove that T can be written as a sum of a semisimple (you can think this as diagonalizable over $\mathbb{C}$) operator S and a nilpotent operator N. Moreover, prove that $SN - NS = 0$.

(3) Let A be a 3×3 matrix. Discuss how one can compute A^{100} with as little computation as possible.

(4) Classify all matrices in $M_n(\mathbb{C})$ up to similarity where $n = 1, 2, 3, 4, 5$.