Jordan Form

1. For all possible shapes of Jordan forms of a $4 \times 4$ matrix, compute the 100th power.

2. Let $S_3$ be the group of permutations of a 3 element set and let $\sigma \in S_3$. Pick a basis $\beta = \{v_1, v_2, v_3\}$ of a three dimensional vectorspace $V$ over a field $F$. Then, sending $v_i$ to $v_{\sigma i}$ defines an automorphism of $V$. Denote this linear operator by $T_{\sigma}$.
   
   (a) Let $F = \mathbb{Q}$. For which $\sigma \in S_3$, is $T_{\sigma}$ diagonalizable? Find all possible Jordan forms.
   (b) Let $F = \mathbb{C}$. For which $\sigma \in S_3$, is $T_{\sigma}$ diagonalizable? Find all possible Jordan forms.

3. Let $A$ be an $n \times n$ matrix whose characteristic polynomial splits. Prove that $A$ and $A^T$ have the same Jordan form.

4. Show that, $e^A$ exists for all square matrices $A \in M_n(\mathbb{C})$. 