

## Parity. Properties

1. Grasshopper jumps along a straight line 2 or 4 cm in either direction. Can it land on a point 17 cm away from starting position after several jumps?
2. Prove that parity of the sum and the difference of two integers is the same.
3. Basil claims that he knows four (integer) numbers such that their sum and their product are odd numbers. Do you know such numbers?
4. Alex wrote the numbers from 1 to 9, one number on each of nine cards. Then he turned the cards down, mixed them and again wrote the numbers from 1 to 9, one number on each card. Then he calculated the sum of the numbers on both sides of each card and multiplied the results. Could he get the odd number?

## Parity. Splitting into pairs

1. 14 people, Liars (who always lie) and Truth Tellers (who always say the truth) sit at round table. Each one said “a person opposite to me is a Liar”. Find the number of Liars and Truth Tellers.
2. Anna and Betty have candies. First, Anna places her candies in a row. Then Betty places her candies, in between of any two adjacent candies. Could all the candies be divided evenly among both girls?
3. Two classes with the same number of the students wrote a test. A principal announced that the number of the students who succeeded was 13 larger then the number of the students who failed. Is the principal’s statement a correct one?
4. During the talks 17 knights from two hostile clans were sitting at round table.  
  
Can it happen that the number of knights who had a friend to his right equals the number of knights who had a foe to his right?
5. Can one draw a broken line consisting of 9 segments such that each segment intersects some other segment exactly once.

- (Slovakia) The seats on a children merry-go-round are numbered in the sequence  $1, 2, 3, \dots$ . On this merry-go-round, Peter was sitting on seat numbered 11, exactly opposite Maria, who was sitting on seat number 4. How many seats are there on this merry-go-round?

### **Parity. Alternating**

- Picking up mushrooms a boy crossed the same railways several times. On which side from his home was the boy when he crossed railway for the 9th time?
- 15 boys and girls sit at round table. Prove that either 2 girls or 2 boys sit together. Prove that there either 2 girls or 2 boys are separated by 2 other children.
- After several moves on a chessboard a Knight returned to its initial position. Prove that the number of moves was even.
- Numbers  $1, 2, \dots, n$  are written in a row. It is allowed to permute any two numbers. Is it possible to return to initial position after 2009 permutations?

### **Parity. Invariant**

- There are three boxes. The first box contains one candy, the second box contains 2 candies, the third box contains 3 candies. You may choose any two boxes and eat a candy from each box. Is it possible to get all boxes empty eventually?
- There are five boxes. There is 1 candy in the first box, 2 in the second, 3 in the third, 4 in the fourth, and 5 in the fifth. You may choose any two boxes and eat a candy from each box. Is it possible to get all boxes empty eventually?
- Seven “0” and one “1” are placed at cube’s vertices. It is allowed to add “1” to both ends of any edge. Is it possible to make all the numbers equal? Operation can be applied as many time as one desires.
- On miracle palm tree grow apples and pears, 13 of each. It is allowed to pick up either one fruit or two fruits at the same time. If any one

fruit is picked up then the same fruit grows instead instantly. If two apples or two pears are picked up then one pear grows instead. If one apple and one pear are picked up then one apple grows instead. Can it happen that in the end only one fruit is left? What a fruit it could be? Can it happen that in the end no fruit is left?

### More problems

1. 15 boys and girls sit at round table. Prove that either 2 girls or 2 boys sit together. Prove that there either 2 girls or 2 boys are separated by 2 other children.
2. 11 boys and girls sit at round table. Can it happen that the number of neighbours “girl-girl” plus “boy-boy” equals the number of neighbours “girl-boy”.
3. Grasshopper jumps along a straight line; first jump is 1 cm, second is 2 cm, and so on. Can it return on its starting position after 25 jumps?
4. Is it possible to list digits from 1 to 9 in such way that between 1 and 2, 2 and 3, . . . , 8 and 9 were placed odd number of digits?
5. There are 3 banana breads. Fillip chooses one and cuts it into 3 or 5 pieces. Then he repeats this procedure several times. Can he get 30 pieces in the end?
6. A set of dominos is placed in a row, according to the rules. On one end is Zero. What can be on the other end?
7. Lena wrote 2009 integers on a board. Can William erase one of the numbers so the sum of the rest was even? What would be the answer in case if Lena wrote 2010 integers?