

Parity

1. During the talks knights from two hostile clans were sitting at round table. It is known that the number of knights who had a friend to his right equals the number of knights who had a foe to his right. Prove that the total number of knights is divisible by 4.
2. There are 30 children in a class who sit in pairs, occupying 15 desks. Exactly half of the girls sit with the boys. Can children be switched into (new) pairs so that exactly half of the boys sit with the girls?
3. 8 rooks are placed on a chess board so no two rooks attack each other. Prove that the number of rooks on black squares is even.
4. A King visited each square of a chess board exactly once and returned to its initial position. Prove that the number of diagonal moves is even.
5. Is it possible to split all integers from 1 to 2010 into several groups so that in each group the maximal integer equals the sum of rest terms in this group?
6. In chess tournament participated 8 teams from 8 different schools. Each game was conducted at one of these school. Can it happen that each team played at every school except its own?
7. Is it possible to place all integers from 1 to 20 at cube vertices and mid-points of edges so that the number at midpoint is average of numbers at the ends.
8. Is it possible to represent 1 as sum of ten fractions in form of $1/m_k$, where m_k some odd integers ($k = 1, 2, \dots, 10$).
9. 31 representatives of 4 aboriginal tribes A, B, C, D sit at fire in a circle. Tribes A and B are enemies, so do tribes C and D . Enemies do not sit together. Prove that two representatives of the same tribe sit together.
10. In one Kingdom live three kinds of the inhabitants: Knights, Princesses and Dragons (100, 99 and 101 respectively). Knights kill Dragons, Dragons eat Princesses, Princesses torment to death Knights. However, a sacred spell makes invincible a someone who kills (eats or torments) an odd number of the inhabitants (unless that someone kills one more).

It is known, that in the end survived only one inhabitant. Who is that inhabitant?

11. King is placed on some square of a chessboard. Each of two players in turns moves it from a square to a square (according to chess rules). It is not allowed to place the King on a square that it was just moved from. The player who is the first to move the King on a square that has been visited wins. Which of the players has a winning strategy?
12. In central cell of a 3×3 table is placed minus; in the rest cells are placed pluses. One can flip all the signs in any column or a row. Can one get a table with all pluses in a finite number of steps?
13. In each of n sectors of a circle is placed a chip (one chip in each sector). It is allowed to move two chips in opposite directions to adjacent sectors. Is it possible to collect all chips at the same sector (by applying this operation several times)? Consider the cases $n = 6$ and $n = 12$.
14. 12 switches are placed in a circular way. Each switch can be either "ON" or "OFF". In one move one can change the positions of any three neighbouring switches. Initially exactly one switch is "ON". Is it possible to get a position with all switches "ON"?
15. Three grasshoppers sit at three vertices of a square. Every second one of grasshoppers (sitting at A) jumps through another one (sitting at B) to the point (C) symmetrical to A with respect to B . Can it happen that at some moment one of grasshoppers lands at the vertex of the square that initially was free?
16. There are two checkers on a straight line: white one to the left of the black. Two operations are allowed:
 - (a) Insertion of two checkers of the same colour and next to each other in any place
 - (b) removal of two checkers of the same colour and next to each other from any place.

Is it possible to get exactly two checkers left: black one to the left of the white in a finite number of steps?