

Online Competition 2008. Juniors (grade 8-9)

1. Several 8 and 7 graders stand in pairs in line. In each pair 8 grader is taller than 7 grader. Is it true that if we rearrange 8 graders and 7 graders according to the height (from the tallest to the smallest) then again in each pair 8 grader is taller than 7 grader?
2. Does there exist a closed broken line consisting of 8 segments so that each segment is intersected by another one and only once.
3. A teacher gave students an assignment:
 - To draw two concentric circles with radii 1 and 10.
 - To the smaller circle, draw three tangent lines so that the points A, B, C of pairwise intersections lie inside of the big circle.
 - To calculate the area S of triangle ABC , as well as the areas S_1, S_2, S_3 of three "sectors" with vertices at points A, B, C .
 - Finally, to compute $S_1 + S_2 + S_3 - S$.

Prove that all students who accomplished the assignment without mistakes got the same answer.

4. Let a and b be positive integers, $a > b$ and $a + b = 2007$. Determine the maximal value of remainder a/b . The same question in case if $a + b = 2008$.
5. A company of 19 aboriginals were sitting at a round table (each aboriginal was either a TruthTeller (always tells the truth) or a Liar (always lies)). Each person declared that both his neighbors were Liars. After a quarrel, some people left the room. Each person who stayed declared that both his neighbors were TruthTellers. "Really, there are no Liars among you now", said the last person who was leaving the room. All who left organized another party at another round table in new place. Each declared that among his neighbors there was exactly one TruthTeller. How many people were remaining in the first room?
6. Two players play in the following game. In equation $x^3 + \square x^2 + \square x + \square = 0$ they put integers in empty boxes: First player places non zero integer, then Second player places integer in any empty box and finally, First player places an integer in the last box. First player's goal is to get an equation with 3 integer roots. Can Second player prevent him?