

## Online Competition 2008. Seniors

1. Among 40 coins identical in appearance 3 are counterfeits (lighter than the real ones). Counterfeit coins weight the same. Separate as many real coins as you can in three weighing by using a simple balance. (Simple balance shows which side is heavier/lighter or both sides in equilibrium).
2. Suppose that  $n$  divides the number  $\underbrace{1 \dots 1}_{n \text{ 1's}}$ . Prove that  $n$  is multiple of 3.
3. Several faces of a convex white polyhedra are repainted into black so that no two black faces share an edge and a total black area exceeds a total white area. Is it possible to inscribe a sphere into this polyhedra ?
4. 1001 rocks are distributed into two piles. An operation consists of moving rocks from the bigger pile to the smaller pile until the number of rocks in smaller pile is doubled. This operation is repeated many times. Is it true that at some moment a smaller pile will contain no more than a) 200 rocks? b) 120 rocks?
5. Solve the system:

$$\begin{cases} x_1 + \sqrt{x_2} = 1, \\ x_2 + \sqrt{x_3} = 1, \\ x_3 + \sqrt{x_4} = 1, \\ \dots\dots\dots \\ x_{25} + \sqrt{x_1} = 1, \end{cases}$$

6. A teacher gave students an assignment:
  - To draw two concentric circles with radii 1 and 10.
  - To the smaller circle, draw three tangent lines so that the points  $A, B, C$  of pairwise intersections lie inside of the big circle.
  - To calculate the area  $S$  of triangle  $ABC$ , as well as the areas  $S_1, S_2, S_3$  of three "sectors" with vertices at points  $A, B, C$ .
  - Finally, to compute  $S_1 + S_2 + S_3 - S$ .

Prove that all students who accomplished the assignment without mistakes got the same answer.