

# MathBattle 1: Season 2003/2004: Problems

December 7, 2003

1. Hydra consists of heads and necks (each neck connects exactly two heads). By one blow of his sword Heracles can cut all the necks connected to any one head. At the same moment, however, new necks grow instantaneously from this head to all the heads, unconnected previously with the head in mention. Heracles defeats hydra, if he separates it into two unconnected parts. Find the minimal number of blows that would allow Heracles to defeat any 100-neck hydra.

2. Solve

$$\begin{cases} x = \sqrt{yz}/(y+z) \\ y = \sqrt{zx}/(z+x) \\ z = \sqrt{xy}/(x+y) \end{cases}$$

3. Quadrilateral with perpendicular diagonals is inscribed into a square (so that each side of the square contains exactly one vertex of the quadrilateral). Each of 8 triangles created is colored in Red or Blue; moreover, triangles that share a side, are painted in different colors. Could it happen that sums of radiuses of circles inscribed into Red and Blue triangles are different?
4. In interval  $(n^2, n^2 + n)$ , where  $n$  is a given positive integer, two distinct positive integers  $A$  and  $B$  are chosen. For the number  $A \cdot B$ , find all positive divisors from the interval in mention
5.  $(10N)$  worms are placed into  $N$  jars. For any two jars with even total number of worms it is allowed to put worms from one jar to the other equalizing numbers of worms in both jars. Applying this operation as many times as one wishes is it always possible to get the same number of worms in all jars?
  - (a)  $N = 8$ .
  - (b)  $N = 10$ .
6. An equilateral triangle  $ABC$  is given. Find a locus of points  $M$ , such that both triangles  $ABM$  and  $ACM$  are isosceles.
7. A segment and a line, parallel to it are given. Using only a straight edge, divide the segment into halves.
8. There is a  $1 \times 2003$  board. Two players in turns place a black or white checker on an empty square of the board. Player, who places a checker next to a checker of different color, loses. If nobody loses, the game is considered to be a draw. Who has a winning strategy? (There are at least 2003 checkers of each color in the stock).
9. It is allowed to make some cuts in  $3 \times 4$  rectangle, so that the figure stays in one piece. Is it possible to wrap up a  $1 \times 1 \times 1$  cube in two layers with this figure?