

MathBattle Superevent: Problems

April 14, 2002

1. Last summer Joe was in the summer camp together with 44 other students. When the camp was over, 950 pairs of students exchanged their addresses. Later Joe wished to write a letter to Jane whose address he did not know. Will Joe always be able to communicate with Jane? In other words: does there always exist a chain of students from Joe to Jane, in which every two consecutive students exchanged their addresses?
2. In the school Chess Club students can play chess either between themselves or with a computer. Last meeting the Club was attended by n students. Each of them played no more than n games, and any two of them who did not play with each other, played altogether no more than n games. Prove that all the students altogether played no more than $n(n+1)/2$ games.
3. There is a set of weights of total weight $2s$. Let's call a natural number k *admissible* if one can chose k weights from the set of total weight s . What is the maximal quantity of admissible numbers when the set consists of 100 weights?
4. The King is placed on some square of the 8×8 chessboard. Two players in turns move it according to the standard rules. It is forbidden to reverse a previous move. The player wins if he moves the King on a square which the King visited before. Who has a winning strategy?
5. The equation $x^3+?x^2+?x+?=0$ is written on the board. The first player names a number and the second replaces one of "?"s by it. The same procedure is repeated twice more. The goal of the first player is to get an equation with three disjoint integer roots. Can the second player prevent him?
6. There are n barrels containing 1 L, 2 L, \dots n L of water. It is allowed to double the amount of water in any barrel taking it from any other barrel which contains enough water. What is the largest quantity of water which could be collected in one barrel?
 - a) $n = 10$
 - b) n is arbitrary
7. The incircle of a triangle divides the segment connecting a vertex with some point on the opposite side into three equal parts. Is it possible for this segment to be the
 - a) altitude
 - b) median
 - c) bisectorof the triangle?
8. Construct triangle if the lengths a and b of two sides are given and if the median drawn to the third side divides the angle in ratio
 - a) 1 : 2.
 - b) 1 : 3.