# MathBattle Superevent: Problems

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**1.** Joe and Jane start moving with constant speeds from the same place in the same direction along straight line. Joe walks and Jane runs. After counting 400 steps Jane turns back. At this moment Joe starts counting his steps and on count of 100 he meets Jane. Whose steps are longer?

**2.** Magician and his Assistant perform a trick. Magician gives a deck of cards numbered from 1 to 78 to Spectator. Spectator chooses any 40 cards and returns them to Magician, leaving the rest of the deck to himself. Magician chooses two cards and gives them to Spectator, who adds one of his cards, shuffles these three cards and passes them to Assistant. Assistant determines which card was added by Spectator. Explain the trick.

**3.** A natural number is written on a blackboard. Two players in turns perform one of the two following operations:

(1) Replace number $n$ by $n-1$

(2) Replace $n$ by $\lfloor \frac{n+1}{2} \rfloor$

The player who first writes the number "1" wins. Which of the players has the winning strategy if the initial number is 1,000,000?

**4.** A point $K$ is chosen on the bisector $AL$ of the triangle $ABC$. It is known that $\angle BKL = \angle KBL = 30°$. Straight lines $AB$ and $CK$ intersect at point $M$ and straight lines $AC$ and $BK$ intersect at point $N$. Find $\angle AMN$.

**5.** There is an airplane with $n$ seats. $n$ passengers with their seats assigned are lined up for the boarding. However, a Crazy Old Lady is the first in the line and she occupies her seat randomly. Then every next passenger entering the plane occupies his/her assigned seat if it is vacant. Otherwise the passenger occupies a seat randomly. Find the probability that the last passenger will occupy his/her seat.

Randomly means that any vacant seat is occupied with an equal probability.

**6.** Find all primes $p$ and $q$ such that $p + q = (p - q)^3$.

**7.** Do there exist five circles on the plane such that any four of them have a common tangent but all five of them do not?

**8.** In the Parliament of Absurdia every MP is affiliated with one of two parties ("Democratic Choice" and "National Block"). It is known that for any two of MPs there exists a third MP which is acquainted with exactly one of them. President of Absurdia can force any group of MPs to change their affiliations; in this case every MP who is acquainted with at least one of MPs from this group, also changes his/her affiliation. No other way to change the affiliation is possible.

Prove that President can make all MPs to be affiliated with Democratic Choice.