MathBattle

January 13, 2001

1. How one can cut a square into 5 pieces such that it would be possible to make three new non-congruent squares using all pieces; no overlapping is allowed.

2. George draw a square $ABCD$ and draw a straight line ℓ passing through B and the midpoint of CD on the blackboard. Then he erased everything but point A and line ℓ. How one can restore the square using only a straightedge and a compass?

3. In a convex quadrilateral ABCD the following is true: $\angle CBD = 2\angle ADB$, $\angle ABD = 2\angle CDB$, $AB = CB$.

   Is it true that $AD = CD$?

4. There is a balance with weights 1,3,5,...,2001 on the left side (Peter's side) and weights 2,4,....,2000 on the right side (Joe's side). Peter starts: he takes off some weights one by one from his side until it becomes (strictly) lighter than Joe's side. Then Joe does the same (downloading his side until it becomes (strictly) lighter thanPeter's side), then Peter takes turn and so on. A player who empties his side first wins.

   Who has the winning strategy?

5. Solve the system:

$$a_1 + a_2 = a_3, a_2 + a_3 = a_4, \ldots, a_{n-1} + a_n = a_1, a_n + a_1 = a_2.$$

6. Find the number of three-digit numbers n such that $n^2 + 8n - 1$ is divisible by 239.

7. In a village some houses are connected by wires. Is it always possible to place a person (either a Truth-teller or a Liar) in each house in such a way that on the question "Are you connected to at least one Liar?" everyone would answer "Yes"?

8. There are 150 lamps in the raw. Some of them are broken. It is known that among each three consecutive lamps at least one is broken. After an electrician fixed some of them, a situation has been changed: now among each four consecutive lamps no more than one is broken.

   What is the minimal possible number of fixed lamps?