MathBattle 1, 2001/2002
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1. There are 101 golden disks of the same appearance with weights 1001 g, 1002 g, ..., 1101 g. An expert knows the weight of each disk and his goal is to prove it to jury for the minimal number of weighings. He has two balances: precise one (shows which cup weights more) and crude one (shows which cup weights more if the difference between weights is more than 1 g; otherwise it shows the balance). Further, he can compare only two disk at the time.

   Which balance an expert should use (he can use only the chosen balance)?

2. There are several pitbulls and sheppards in one dog pond. Yesterday every pitbull was in quarrel with the same number of sheppards while each sheppard was in quarrel with the different number of pitbulls. Today some of them befriended and the situation reversed: every sheppard is in quarrel with the same number of pitbulls while each pitbull is in quarrel with the different number of sheppards. Find the number of dogs of each breed.

3. The King of Canniballia invited all the Cannibals of his Kingdom for a feast. However, some of Cannibals would like to eat other Cannibals instead (If A would like to eat B, it is not necessary that B has a reciprocal feeling). It is given that each "food chain": A wants to eat B, B wants to eat C, and so on, consists of no more than 6 Cannibals. King wants to place them in a several rooms such that nobody has gastronomical feeling to his roommates. What is the minimal number of rooms, which is enough for sure?

4. There is a tetrahedron with a volume V and a surface area S. The planes of its faces were moved in the normal direction at the distance h outwards. Find the volume and the surface area of the new tetrahedron.

5. Peter claims that any six consequtive numbers could be placed instead of ? in such a way that the following system

$$\begin{cases} ?x + ?y = ? \\ ?x + ?y = ? \end{cases}$$

   has integer solutions.

6. Product of the root of equation $ax^2 + bx + b = 0$ and the root of equation $ax^2 + ax + b = 0$ is equal to 1. Find these roots provided $a \neq 0$, $b \neq 0$.

7. There are 2001 points on the plane. Two players in turn connect some pair of them by an edge. The player who draws the third edge from some vertex, looses. Which player has a winning strategy?

8. Let A', B', C', D', E', F' be midpoints of sides AB, BC, CD, DE, EF, FA of the convex hexagon $ABCDEF$. Find an area of hexagon if areas of triangles $\triangle ABC'$, $\triangle BCD'$, $\triangle CDE'$, $\triangle DEF'$, $\triangle EFA'$, $\triangle FAB'$ are given.