SOAR MathBattle

1. (**Captain' contest**) Two line segments drawn from vertex A divide side BC of triangle ABC into 3 equal parts. Is it possible that they also divide angle A into 3 equal parts?

2. (**Audience problem**) The infinite grid paper was painted except a 7×7 square. Peter paints cell by cell in such a way that on each step the newly painted cell has exactly one painted neighbour (by side). What is the maximal number of cells Peter can paint?

First Day Problems

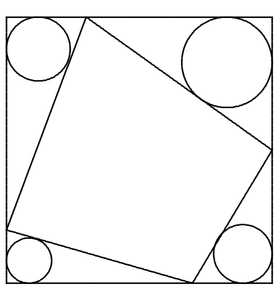
1. Altitude CD and bisector AE of right-angled triangle ABC ($\angle C = 90°$) intersect at point F. Let G be the point of intersection ED and BF. Prove that the area of quadrilateral $CEFGF$ is equal to the area of triangle BDG.

2. Given that $\triangle ABC$ is acute with $\angle A = 40°$ and $\angle CMB = 110°$ for some interior point M. The perpendicular bisectors to BM and CM intersect AB and AC at points P and Q respectively. Prove that point M belongs to the line segment PQ.

3. There are 101 straight lines on the coordinate plane. All their intersection points are marked. Is it possible that every straight line contains exactly 50 marked points with positive abscises and 50 with negative?

4. Point P is marked on the side AB of the parallelogram $ABCD$. Construct inscribed parallelogram with a vertex P such that it cuts from $ABCD$ four triangles of equal areas.

5. (**Reserve problem**) A quadrilateral is inscribed into a square with the side 1. The sides of the quadrilateral are hypothenuse of four right-angled triangles. Prove that the sum of the radii of incircles of these triangles does not exceed $2 - \sqrt{2}$ and that equality holds iff the sides of the quadrilateral are parallel to the diagonals of the square.



Second Day Problems

1. (**Either (a) or (b)**) Let AA_1 and BB_1 be altitudes of acute triangle ABC.

   (a) Let D be a point on AA_1 such that $A_1D = B_1D$. Let E be a midpoint of AB. Prove that A, B_1, D, E belong to the same circumference.

   (b) Let K and M be midpoints of AB and A_1B_1 respectively. Let L be an intersection of line segments AA_1 and KM. Prove that A, B_1, K, L belong to the same circumference.

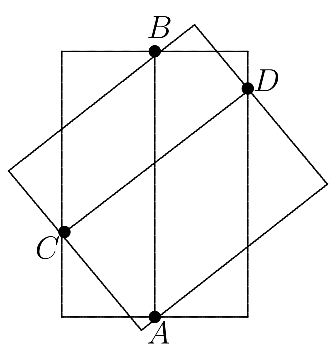
2. Two line segments of the lengths m and c are given. construct the right-angled triangle for which c is a hypothenuse and m is the sum of two legs, using a compass and a ruler.

3. There is bacteria in one cell of the infinite grid paper (1-st generation). In a second two more bacteria of the second generation appear in any two neighbouring cells (by side). Every second each bacteria of the latest generation gives birth to exactly two new bacteria of the next generation occupying two empty neighbouring cells. No two bacteria can occupy the same cell.

   (a) What is the maximal number of generations?

   (b) The same question if “neighbouring” means also by vertex.

4. (**or # 5**) Red and blue rectangles are intersected. It happens that the segment AB is parallel to the side of the blue rectangle and the segment CD is parallel to the side of the red rectangle. Is it true that the areas of both rectangles are always equal?



5. Is it true that one can cut any quadrilateral into three trapezoids?