

# ✠ MATHBATTLE 3 ✠

• U.T.S. - Mixed Team •

April 1, 2001

1. Is it possible to cut a circle into seven parts of equal area by three straight cuts?
2. In a community every night one of the members gives a party to all he knows and after party all the guests know each other. After everybody gave at least one party, there are two people left who don't know each other. Prove that they cannot be introduced one another at the next party.
3. There are three piles of rocks. Sisyphus carries rocks from one pile to another taking one rock every time. Every time he is paid by the number of coins equal to the difference between the number of rocks in a pile in which he puts the rock and the number of rocks in a pile from which he took it (the rock itself does not count). If the difference is negative he pays back (he accumulates the debt). After a while it happened that all rocks returned to their original piles. What is the largest possible profit of Sisyphus?
4. Is it possible that all the integers from 1 to 100 are terms of twelve geometric series?
5. The sequence of natural numbers  $a_i$  is such that for all  $i \neq j$   $\text{lcm}(a_i, a_j) = \text{lcm}(i, j)$ . Prove that  $a_i = i$  for all  $i$ .
6. The "Complete Rabbit Almanac" consists of 10 volumes. They are put on a shelf ulmost in order: each volume is either on its own place or on an adjacent place. How many such arrangements are possible?
7. There are  $2n + 1$  cows. It is known that if any cow is removed then all the others cows can be divided into two groups of equal numbers and equal masses. Assuming that weight of any cow is an integer prove that all the cows have equal masses.
8. Can one cover a cube of edge 1 by six parallelograms each of the area 1 and different from a rectangle?
9. Let  $a, b, c$  be sides of a triangle. Prove the following inequality:

$$\frac{a^2 + 2bc}{b^2 + c^2} + \frac{b^2 + 2ca}{c^2 + a^2} + \frac{c^2 + 2ab}{a^2 + b^2} > 3.$$

10. In  $\triangle PQR$  is given that  $\angle PQH = \angle HQM = \angle MQR$ , where  $QH$  and  $QM$  are its altitude and median respectively. Find all possible values of  $\angle PQR$ .