✠ MATHBATTLE 2 ✠

♡ Earl Haig Team A – Earl Haig Team B ♠

February 11, 2001

1. Find a triangle $\triangle ABC$ with base AB equal to c and altitude from C equal to h such that the product of its altitudes is maximal.
2. There are 99 coins of identical appearance. One of the coins is counterfeit, and it is lighter than the real ones. Is it possible to find out which coin is counterfeit if only seven weighings are allowed and each coin cannot be weighted more than twice?
3. Evaluate the integral $\int_0^\pi (|\sin 2000x| - |\sin 2001x|)dx$.
4. Let $f(x) = x^2 + 12x + 30$. Solve the equation:
$$f(f(f(f(f(x))))) = 0.$$
5. Does there exist a tiling of the plane with congruent pentagons?
6. Several students are sitting at a round table. The teacher walks clockwise around the table and hands out nuts to some of them according to the following rule. He selects one child and gives him a nut, then he skips the next child and gives a nut to the next one, then he skips 2 and gives a nut to the next one, then he skips 3, and so on. What are the possible number of students so that eventually, perhaps after many rounds, all the students will have at least one nut each?
7. Peter and Anna are playing the following game. Peter thinks of a code, i.e., a sequence of natural numbers $(a_1, a_2, \ldots, a_{2001})$. To break the code Anna gives a sequence of natural numbers $(b_1, b_2, ..., b_{2001})$ and Peter tells her the value of $a_1b_1 + a_2b_2 + \cdots + a_{2001}b_{2001}$. How many turns it will take for Anna to break the code, assuming that she is very smart?
8. A hiker walked for 3.5 hours. For any one hour period he walked exactly 5 km.
Is it true that his average speed was 5 km/h?
9. There are two nails in the wall and a picture with a string attached to it (in a usual way: each end of the string is attached to one corner of the picture). Is it possible to hang this picture in such a way that it would fall down if either of the nails is removed?
10. In "Guinness world records 2000 Millennium edition" on page 167 it says that the largest possible prime number known is $23021^{377} - 1$.
Find any of its divisors (different from 1 and itself).