MathBattle 10: Problems

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1. Planar convex figure is bounded by a circular arc DB and two segments AB and AD. Construct a straight line which divides into halves

a) perimeter of the figure;

b) area of the figure;

2. Let a, b, c be real numbers such that $(a + b + c)c < 0$. Prove that $b^2 > 4ac$.

3. There is a circular hole somewhere in a square-shaped golf field. A grass-hoper hops on this field: each time it chooses a vertex and hops in its direction covering exactly the half of the distance. Can he reach the hole?

4. *Graph* is a set of *vertices*; some of them are connected by *edges*. Each edge connects exactly two vertices. We call the coloring of the vertices (graph) *regular* if no two vertices of the same color are connected by an edge. Let us consider a graph regularly colored into k colors such that it cannot be regularly colored into $k-1$ colors. Prove that there exists a path containing k vertices, all of them of different colors.

5. Seven cyclists, each starting at different times, rode in one direction. One of them had a waterbottle. From time to time, one cyclist got ahead of another. If one of them had a bottle he gave it to the other one. The bottle was not transfered in any other way. What is the minimal number of times when somebody got ahead of someone else if every cyclist had the bottle at least once?

6. There is a frame obtained from a rectangular $m \times n$ $(m \geq 3, n \geq 3)$ board by removing all interior squares. Two players cut it in turns. In one move it is allowed to cut out any number of squares in row from the same side of the frame provided that the remaining part does not split into two pieces. The player who makes the last move wins. Who has a winning strategy?

6*. Two coiners play the following game: in turns they issue a new coin of integer value. It is not allowed to issue a coin of value 1 or a coin already issued or a coin which could be exchanged by any number of coins already issued. The coiner who cannot issue a new coin loses.

a) Prove that the game cannot continue infinitely.

b) Who has a winning strategy?

7. A set (possibly with repetitions) of 2001 numbers satisfies the following condition: if every number is replaced by the sum of all other numbers, then the set does not change. Find the product of all numbers in the set.

8. 49 buttons are arranged in 7×7 square. Every button can be either ON or OFF. Pressing each button we change the states of all buttons in 3×3 square with the center at this button. Is it possible to have all 49 buttons off simultaneously??

9. Find a seven digit number such that its first digit equals to a number of "0" in it, its second digit equals to a number of "1" in it, and so on,