✠ MATHBATTLE ✠

December 10, 2000

1. A set of 10 coins, identical in appearance, is arranged in a row. It is known, that any real coin (10 g) is on the left to any counterfeit coin (9 g). Is it possible to separate real coins from fake ones using two weighings? (The balance only shows which side is heavier).

2. Find three triangles so that one could construct (using all three pieces and without overlapping) each of three figures:
   (a) a triangle
   (b) a convex quadrilateral
   (c) a convex pentagon.

3. Point E is given on the diameter AC of the circle. Draw a chord BD through E in such a way that an area of the quadrilateral $ABCD$ is maximal.

4. Nine flies are crawling on a sphere with the radius 1. Prove that at any moment there are two flies such that the distance between them does not exceed the square root of 3.

5. Initially, on each of the first 500 steps of a 1001-step staircase there is a stone. S. and A. take turns starting with S. as follows:

   S. picks up any stone and carries it up to the next free step. Then, A. rolls one step down any stone if the previous step below it is free.

   S. tries to carry up a stone to the very top. Can A. prevent him?

6. There is an infinite number of hit men. Each hit man tries to kill exactly one of the rest. Prove that there exists an infinite subset of hit men such that nobody tries to kill anyone in that group.

7. Is the number $2^{10} + 5^{12}$ prime or composite?

8. It is known that $m^2 + n^2 + m$ is divisible by mn for some positive integers m and n. Prove that m is a perfect square.

9. Square is cut into 100 squares, 99 of which have side 1. What are the possible values of the original square area?

10. There is a rectangular cake with a rectangular chocolate bar on it. How one can divide evenly both cake and bar by one straight cut?