

ENCOURAGING YOUR KIDS TO LIKE AND STUDY MATH

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GENERAL STRATEGY

- Encourage your child to calculate in the head, rather than by calculator. Do not push to memorize multiplication table but rather inspire to make these operations in the head until the table will naturally be memorized.
- Discuss logics and math problems together. Even if the solution is not found by effort of the whole family, your child would benefit from seeing the problem from different angles and exploring different approaches.
- Encourage your child to play chess, board games, solve puzzles, and mazes. It develops strategical thinking, strengthen memory, expand imagination.
- Keep in mind that pushing children to earn better grades often has negative effects. By trying to obtain a higher mark and be nice to the teacher and to parents, children are “following directions” rather than developing their own way of thinking.

- Praise children for their achievements, but do not criticize for the insufficient performance. It may kill any interest in mathematics.
- Do not rush your children to follow the senior grades curriculum. The bare knowledge of advanced material does not make children “talented”. Developing ability to think is more important.
- Go into many directions. Study accessible material, not covered by a school curriculum. Geometry, logic, olympiad types of problems are the best choice. They develop ability to think and to prove. Any aspiring 5th grader should know the Pythagorus theorem, and be able to solve the simple equations (or even more complicated ones like quadratic equations).

- **Thinking deeply is more important than thinking quickly.** Very often teachers and parents encourage children to solve problems quickly. To think fast shows ability of brain to process information and react. This is crucial for many professions. However, to think quickly does not mean to think deeply. The ability to think deeply and make connections is much more important for mathematicians and researchers.

Great mathematician David Hilbert when he was at school was slow in solving easy problems. He usually finished the easy assignments among the last in his class. When assignments were harder, he was among the first students in class. Furthermore, in case of really hard problems, which no one in the class could solve, he was delivering solutions with the same speed as he was solving easy problems.

- **Solve problems from the previous contests.** One can find problems from the previous competitions (Math Kangaroo, Abacus, Pythagorus contests, Waterloo contests). Older and more advanced children may tackle the problems of different mathematical olympiads (Tournament of Towns, CMO, USAMO, APMO, IMO).

LOGICAL PROBLEMS FOR THE WHOLE FAMILY

CROSSING THE RIVER

Two men came to a river. There was one small boat such that only one man can use it safely. However, both men crossed the river. Explain, how it could be.

TWO PURSES

Two purses together contain two coins while one of the purses contains twice more coins than another. Explain, how it could be.

APPLEBASKET

There are 3 apples in applebasket. Divide them equally between two friends and yourself so that each of you get an apple and still one apple would be left in the basket.

HAIRDRESSER

Why a hairdresser in Geneva is more eager to shave two Frenchmen than one German?

CLAIRVOYANT

Alex bet that he knew the score in a football game between teams A and B before the match had been started. He won the bet. What was the score he claimed?

A CANNONBALL AND A WALL

What happens if all-crashing cannonball crashes into an indestructible wall?

TWO TRAINS

A train departs from Toronto for Montreal. One hour later, another train departs from Montreal for Toronto. Both trains move with the same speed. When they meet, which train will be closer to Montreal?

WOLF, GOAT AND CABBAGE

A man with wolf, goat and cabbage needs to cross a river. A boat could carry only the man and one animal or the man and the cabbage. Find a way for the group to cross the river. (Note that in absence of man, wolf devours goat and goat eats cabbage).

PLATOON

A platoon of soldiers needs to cross a river. The soldiers found two boys sailing on a small boat. The boat is so small that only two boys or one adult can use it safely. Is there a method for the platoon to cross the river?

MISSIONERS AND CANNIBALS

Three missionaries and three cannibals need to cross a river. A boat can carry only two men at a time. Missioners are afraid to stay with cannibals on the shore if they are minority in the group. Only one of the missionaries and one of the cannibals know how to sail. Find the way how the group can cross the river.

BRIDGE

A family of four: a man, a woman, a boy, and an old man need to cross a river during the night. Small bridge can be safe for two people at the time. The family has one flashlight with them. It takes one minute, two minutes, 5 minutes and 10 minutes for the man, the boy, the woman, and the old man respectively to cross the bridge. Going in pairs, they go with minimal speed of two. Find the minimal time they need to cross the river.

FISHING

Nicolas and his son and Peter and his son were fishing. Nicolas and his son caught the same number of fish while Peter caught three times more than his son. Together, they caught 25 fish. How many fish did Nicolas catch?

THE ROPE

There is a rope of 1 and half meter long. Without any measuring instruments cut exactly 1 meter of rope.

TWO ROPES

There are two ropes. It is known that each rope burns out completely in exactly 1 hour. However, the pace of burning is not uniform. How one can measure 45 minutes interval, using these ropes and a box of matches?

HOURLASSES

There are two hourglasses: for 7 and 11 minutes. An egg has to be boiled 15 minutes. Using hourglasses measure the exact time.

FRY PAN

A cutlet should be fried for 4 minutes, 2 minutes on each side. There is a fry pan enough for two cutlets. How to fry 5 cutlets as fast as possible?

THE BRICK

A brick weights 2 kg plus one third of its weight. How much does it weight?

TWO PAILS

One needs to bring exactly 6 litres of water from river. There are only 2 pails available, one pail of 4 litres volume and another of 9 litres. How it can be done?

THREE COINS

Among three coins identical in appearance one is counterfeit (lighter than the real one). Using a simple balance, separate the counterfeit coin in one weighing. (A simple balance shows whether both sides of the balance are in equilibrium or one of the sides is lighter/heavier.)

FOUR COINS

Among four coins denomination of 1 kopeika, 2 kopeikas, 3 kopeikas and 5 kopeikas one is counterfeit: its weight in grams does not correspond to its denomination (the weight of the real coin corresponds to its denomination). In two weightings separate the counterfeit coin.

TWO INEPT COINERS

Jack and Jim are two coiners: Jack's coins are heavier than the real ones while Jim's coins are lighter. Among 10 coins one is counterfeit. In two weightings define who of the coiners produced it, Jack or Jim?

SURVIVOR ISLAND

In one Kingdom live three kinds of the inhabitants: Knights (10), Princesses (9) and Dragons (11). Knights slay Dragons, Dragons devour Princesses, Princesses torment to death Knights. However, the one who has slain, devoured or tormented an odd number of the creatures becomes invincible (until he or she kills the next victim). It is known, that in the end only one inhabitant survived. Who is this lucky survivor?

THREE WISE MEN

The King summoned three wise men and shown them 2 black and 3 white hats. Then he put hats on each of them so each wise man could see the colours of the hats of two others but not his own. Then the King asked “What is the colour of your hat?” but nobody answered. The King repeated his question and still nobody answered. But when the King asked the same question the third time all three wise men answered correctly. Which hats the King put on their heads?

SOLUTIONS TO PROBLEMS

CROSSING THE RIVER

Men came from the different sides of the river.

TWO PURSES

One purse is inside of the other.

HAIRDRESSER

Two Frenchmen pay twice more than one German.

CLAIRVOYANT

Before the game started, the score was 0:0.

A CANNONBALL AND A WALL

Both such objects can not exist at the same time: existence of one contradicts to the existence of the other.

TWO TRAINS

When they meet, both trains are on the same distance from any city.

WOLF, GOAT AND CABBAGE

- 1 Man goes with the goat, leaves the goat and comes back;
- 2 Man takes the wolf, leaves the wolf but takes the goat back;
- 3 Man leaves the goat but takes the cabbage, which he leaves with the wolf;
- 4 Man takes the goat.

PLATOON

Let us call boys Adam and Ben.

- Adam is left on the opposite shore, while Ben sails to the shore where platoon is;
- One adult sails to the opposite shore, Adam comes back to pick Ben;
- Adam is left on the shore with the soldier, Ben returns to the shore with platoon;
- ...
- This procedure continues until the last adult crosses the river.

MISSIONERS AND CANNIBALS

Let M, m, m and C, c, c be Missioners and Cannibals (M and C the ones who can sail)

- ① C, c ; C is back
- ② C, c ; C is back
- ③ M, m ; M, c are back
- ④ M, C ; M, c are back
- ⑤ M, m ; C is back
- ⑥ C, c ; C is back
- ⑦ C, c .

BRIDGE

It is easy to check by observation that the best scenario takes 17 minutes.

- 1 The father and the boy: the father is coming back;
- 2 The old man and the mother; the boy is coming back;
- 3 The father and the boy.

FISHING

Assumption that Nicolas and his son and Peter and his son are different people leads to contradiction (in this case the total number of fish should be even). Thus, we assume that we have 3 people (3 generations). It can be N-P-S (in this case there is no solution either) or it can be P-N-S. If S (Nicolas' son) caught x fish, then Nicolas also caught x fish, while Peter caught $3x$ fish. Together, they caught $5x$ fish, which equals 25. Therefore, $x = 5$ (the number of fish Nicolas caught).

THE ROPE

Fold it into 3 lines.

TWO ROPES

Ignite one rope on one end and the second rope on both ends. When the second rope is burned, ignite the first rope on the other end.

HOURLASSES

Set both hourglasses. In 7 minutes turn over 7 minutes hourglasses. After 4 more minutes (11 minutes from start) turn over 7 minutes glasses again. The total times till 7 minutes glasses are empty is $11 + 4 = 15$ minutes.

FRY PAN

Let a, b, c, d, e be the cutlets. The sequence of frying:

- ① First two minutes: a, b
- ② Second two minutes: a (on the second side), c
- ③ Third two minutes: b, c (both on the second side)
- ④ Last four minutes : d and e (both sides)

Total time: 10 minutes

THE BRICK

Note that 2 kg constitutes $\frac{2}{3}$ of the brick. Thus the brick weights 3 kg.

TWO PAILS

- 1 fill 9 litre pail with water;
- 2 pour out 4 litres of water into 4 litres pail (thus 5 litre of water is left in 9 litres pail);
- 3 dispose water from 4 litres pail;
- 4 pour out 4 litres of water into 4 litres pail (thus 1 litre of water is left in 9 litres pail);
- 5 dispose water from 4 litres pail;
- 6 pour out 1 litres of water into 4 litres pail;
- 7 fill 9 litres pail with water;
- 8 pour out water into 4 litres pail until it is full (3 litres). Thus $9 - 3 = 6$ litres of water is left in 9 litre pail.

THREE COINS

Weight any 2 coins. In the case of equilibrium the counterfeit coin is one put aside; otherwise, we choose the coin which is lighter.

FOUR COINS

Digits below stand for coins of the given denominations.

- ① $\underline{2 + 3} ? \underline{5}$ (weighting coins 2 and 3 vs 5)
 - if $2 + 3 = 5$ the counterfeit is 1.
 - if $2 + 3 < 5$ (then either 2 or 3 is lighter or 5 is heavier).
- ② $\underline{1 \text{ (real)} + 2} ? \underline{3}$;
 - if $1 + 2 = 3$ (then 5 is counterfeit).
 - if $1 + 2 < 3$ the counterfeit is 2 (can not be 3 since counterfeit is lighter).
 - if $1 + 2 > 3$ the counterfeit is 3 (since counterfeit is lighter).

The case $2 + 3 > 5$ can be considered by using the similar arguments.

TWO INEPT COINERS

3?3 (4 coins are put aside; we call them “the group”).

- 1 if $3=3$ than all these coins are real. Take any 4 of them and weight against the group put aside. If the group is heavier, the counterfeit is produced by Jack; otherwise, it is produced by Jim.
- 2 If $3 \neq 3$ (say $3 < 3$) let us choose the heavier group and compare with 3 of real ones (which were put aside).
In the case of equilibrium the counterfeit is in lighter group, so it is produced by Jim; otherwise, it produced by Jack.

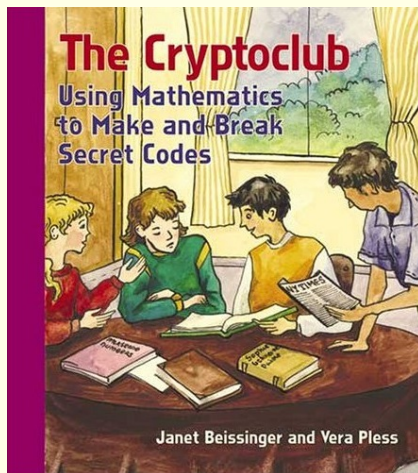
SURVIVOR ISLAND

- 1 Assume that the lone survivor is a Knight. This implies that all Princesses are devoured. It can be only in case if each Princess tormented an even number of Knights. Thus, the total number of Knights alive must be even. Contradiction.
- 2 Assume that the lone survivor is a Princess. Then all Knights are dead. This implies that each of them slayed an even number of Dragons. Therefore, the total number of killed Dragons is even. Contradiction.
- 3 Thus, the lone survivor must be a Dragon. This is possible: for example, one Knight slays 10 Dragons, one Princess torments to death all Knights. The remaining Dragon devours all Princesses. Bon Appétit!

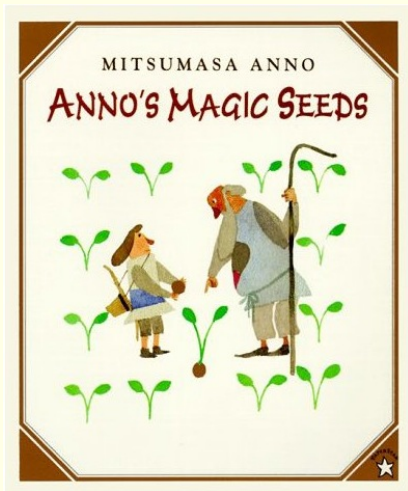
THREE WISE MEN

- 1 Note that if two wise men were in a black hats then the third one would give the answer after the first time the King asked the question. Since nobody answered then this case is eliminated. Now everyone knows that there is no more than 1 black hat.
- 2 If someone was in a black hat, then the other two wise men would see it and since they already know that there is no more than one black hat, these two wise men would answer the second time the King asked the question.
- 3 Since nobody answered then everyone knows that there are no black hats at all. Therefore, the third time all three wise men gave the answer.

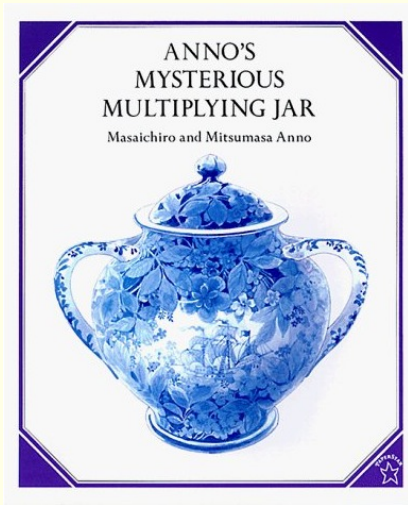
BOOKS (RECOMMENDED BY ED BARBEAU)



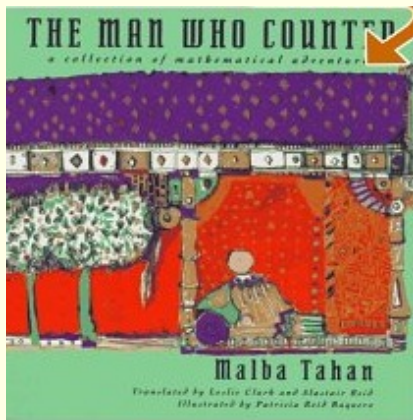
Janet Beissinger & Vera Pless,
The Cryptoclub: using mathematics to make and break secret codes



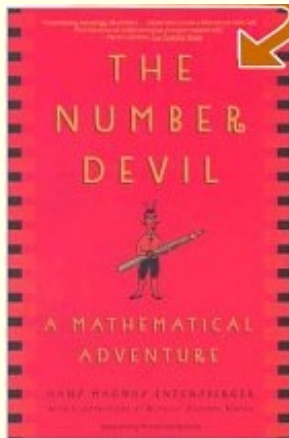
Mitsumasa Anno
Anno's Magic Seeds



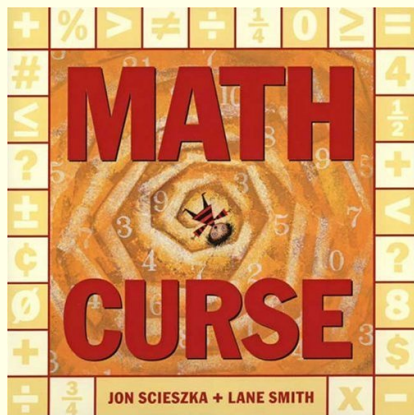
Mitsumasa Anno
Anno's Mysterious Multiplying Jar



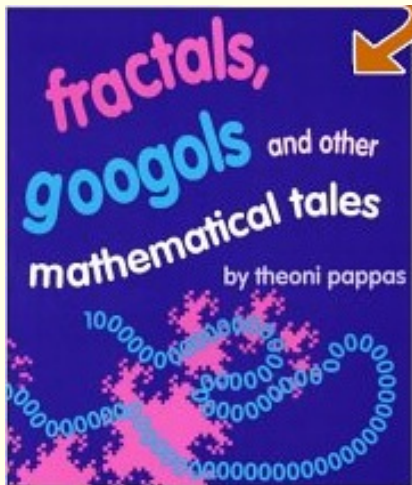
Malba Tahan
The Man Who Counted



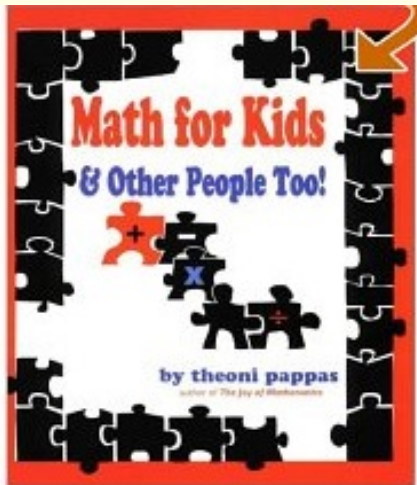
Hans Magnus Enzensberger
*The Number Devil:
a Mathematical Adventure*



Jon Scieszka
The Math Curse

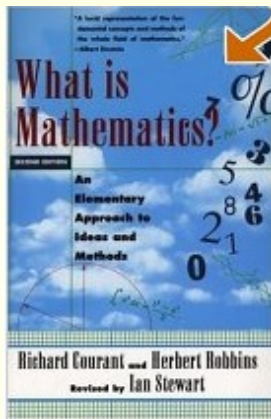


Theoni Pappas
*Fractals, Googols and other
 Mathematical Tales*

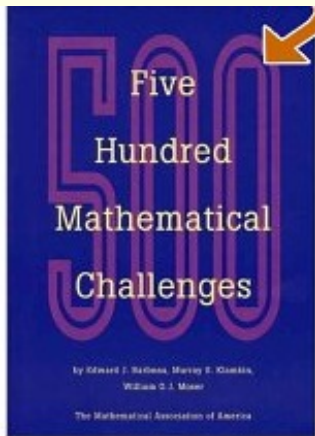


Theoni Pappas
*Math for Kids & Other People
 Too!*

MORE BOOKS



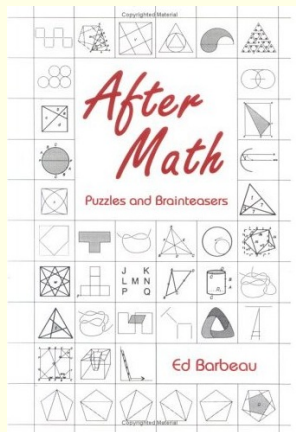
Courant, Robbins
What is mathematics?
(Recommended for grade 8 and above)



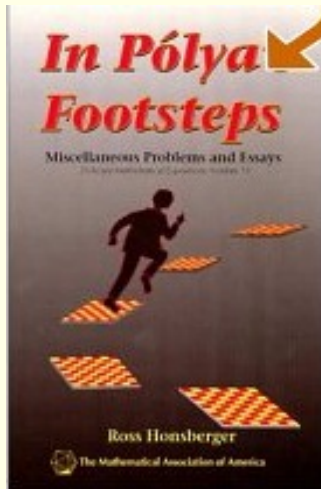
Barbeau, Klamkin, Moser
Five Hundred Mathematical Challenges
(Recommended for grade 8 and above)



Ed Barbeau
Power Play
 (For grade 9 and above)

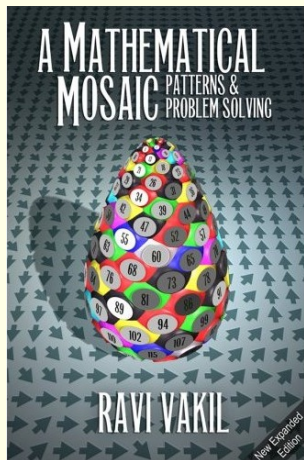
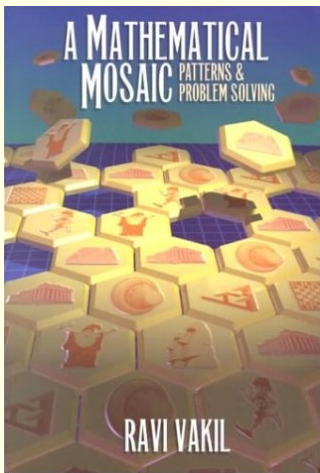


Ed Barbeau
After Math
 (For grade 6 and above)



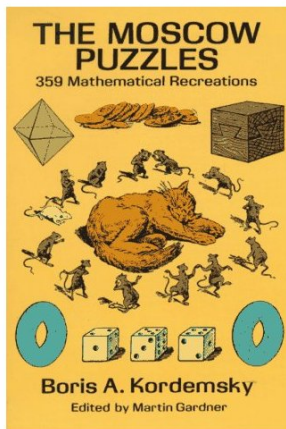
Honsberger

In Pólya's Footsteps (starting from grade 9)

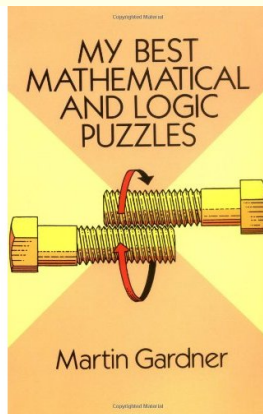


Ravi Vakil

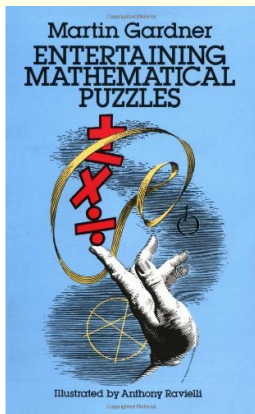
A mathematical Mosaic (starting from grade 7)



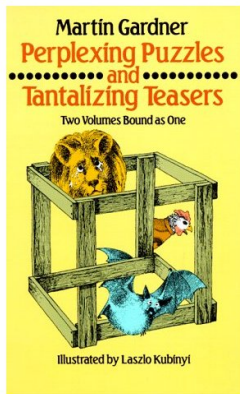
B. Kordemsky,
The Moscow puzzles



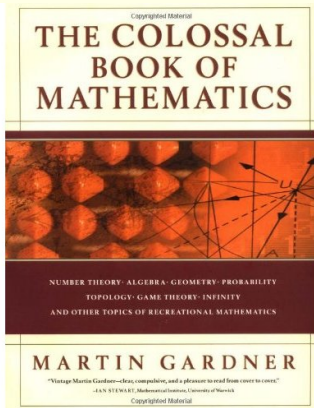
M. Gardner,
*My best mathematical and logic
puzzles*



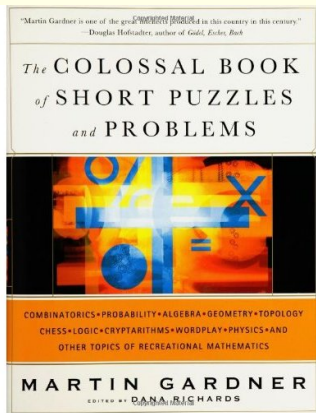
M. Gardner,
*Entertaining mathematical
puzzles*



M. Gardner,
*Perflexing puzzles and
Tantalizing teasers*



M. Gardner,
*The colossal book of
Mathematics*



M. Gardner,
*The colossal book
of short puzzles and problems*

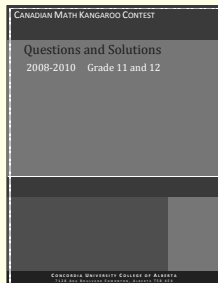
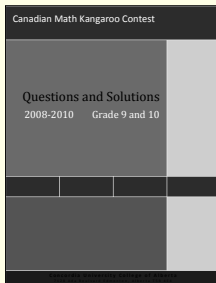
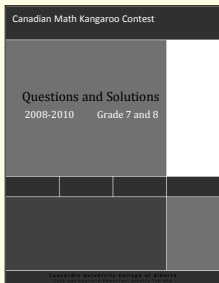
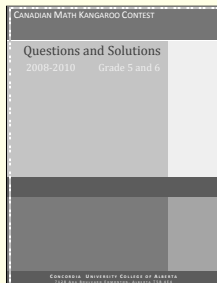
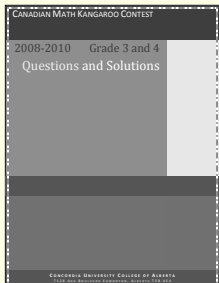
KANGAROO CANADA: BOOKLETS

A group of volunteers, professors and students, prepared booklets for preparations to Math. Kangaroo, divisions 3–4, 5–6, 7–8, 9–10, and 11–12. Each booklet includes problems, answers and solutions for years 2008, 2009, 2010.

Authors donate their work to sponsor Math Clubs and related activities for participants of Math Kangaroo.

They are organized as follows:

Year 2008	1
Year 2008 Grade 7-8 Questions	2
Year 2008 Grade 7-8 Answers	5
Year 2008 Grade 7-8 Solutions	6
Year 2009	11
Year 2009 Grade 7-8 Questions	12
Year 2009 Grade 7-8 Answers	15
Year 2009 Grade 7-8 Solutions	16
Year 2010	21
Year 2010 Grade 7-8 Questions	22
Year 2010 Grade 7-8 Answers	25
Year 2010 Grade 7-8 Solutions	26



Cost of a single booklet: \$30.00. All proceedings go towards organizations of Math Competitions and Math Clubs.

How to pay and pick up: Please note only cheques or money orders will be accepted, no cash. The cheque should be written:

CHEQUE

Payable to: Department of Mathematics, University of Toronto

Memo: for MK booklet, and indicate division

Do not forget amount, date and signature.

PICK-UP

University of Toronto, Department of Mathematics,
(40 St. George Str., Bahen Centre for Information Technology, 6th floor),
Reception Desk (room BA6290);

ask Patrina Seepersaud.

Time: anytime from 9:30 to 12:00 and from 13:00 to 4:30, Monday–Friday.