International Mathematics

TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Spring 2015**

1 [**3**] Pete summed up 100 consecutive powers of two, while Basil summed up several first consecutive positive integers. Can they get the same result?

2 [**4**] A moth made four small holes in a square carpet with a 275 cm side. Can one always cut out a square piece with a 1 m side without holes? (Consider holes as points).

3 [**5**] Among $2n+1$ positive integers there is exactly one 0, while each of the numbers $1, 2, ..., n$ is presented exactly twice. For which n can one line up these numbers so that for any $m = 1, \ldots, n$ there are exactly m numbers between two m's?

5 [**5**] Points K and L are marked on the median AM of triangle ABC, so that $AK = KL = LM$. Point P is chosen so that triangles KPL and ABC are similar (the corresponding vertices are listed in the same order). Given that points P and C are on the same side of line AM, prove that point P lies on line AC.

5 [**5**] 2015 positive integers are arranged in a circular order. The difference between any two adjacent numbers coincides with their greatest common divisor. Determine the maximal value of N which divides the product of the numbers, regardless of their choice.