

**37th International Mathematics  
TOURNAMENT OF THE TOWNS**

**Senior O-Level Paper**

**Fall 2015**

- 1 [3] Let  $p$  be a prime number. Determine the number of positive integers  $n$  such that  $pn$  is a multiple of  $p + n$ .
- 2 [4] Suppose that  $ABC$  and  $ABD$  are right-angled triangles with common hypotenuse  $AB$  ( $D$  and  $C$  are on the same side of line  $AB$ ). If  $AC = BC$  and  $DK$  is a bisector of angle  $ADB$ , prove that the circumcenter of triangle  $ACK$  belongs to line  $AD$ .
- 3 [4] Three players play the game “rock-paper-scissors”. In every round, each player simultaneously shows one of these shapes. Rock beats scissors, scissors beat paper, while paper beats rock. If in a round exactly two distinct shapes are shown (and thus one of them is shown twice) then 1 point is added to the score of the player(s) who showed the winning shape, otherwise no point is added. After several rounds it occurred that each shape had been shown the same number of times. Prove that the total sum of points at this moment was a multiple of 3.
- 4 In a country there are 100 cities. Every two cities are connected by direct flight (in both directions). Each flight costs a positive (not necessarily integer) number of doubloons. The flights in both directions between two given cities are of the same cost. The average cost of a flight is 1 doubloon. A traveller plans to visit any  $m$  cities for  $m$  flights, starting and ending at his native city (which is one of these  $m$  cities). Can the traveller always fulfil his plans given that he can spend at most  $m$  doubloons if
- a [2]  $m = 99$ ;
- a [2]  $m = 100$ ?
- 5 [5] An infinite increasing arithmetical progression is given. A new sequence is constructed in the following way: its first term is the sum of several first terms of the original sequence, its second term is the sum of several next terms of the original sequence and so on. Is it possible that the new sequence is a geometrical progression?