

**37th International Mathematics
TOURNAMENT OF THE TOWNS**

Junior O-Level Paper

Fall 2015

- 1 [4] Is it true that every positive integer can be multiplied by one of integers 1, 2, 3, 4 or 5 so that the resulting number starts with 1?
- 2 [4] A rectangle is split into equal non-isosceles right-angled triangles (without gaps or overlaps). Is it true that any such arrangement contains a rectangle made of two such triangles?
- 3 [5] Three players play the game “rock-paper-scissors”. In every round, each player simultaneously shows one of these shapes. Rock beats scissors, scissors beat paper, while paper beats rock. If in a round exactly two distinct shapes are shown (and thus one of them is shown twice) then 1 point is added to the score of the player(s) who showed the winning shape, otherwise no point is added. After several rounds it occurred that each shape had been shown the same number of times. Prove that the total sum of points at this moment was a multiple of 3.
- 4 [5] In a right-angled triangle ABC ($\angle C = 90^\circ$) points K , L and M are chosen on sides AC , BC and AB respectively so that $AK = BL = a$, $KM = LM = b$ and $\angle KML = 90^\circ$. Prove that $a = b$.
- 5 In a country there are 100 cities. Every two cities are connected by direct flight (in both directions). Each flight costs a positive (not necessarily integer) number of doubloons. The flights in both directions between two given cities are of the same cost. The average cost of a flight is 1 doubloon. A traveller plans to visit any m cities for m flights, starting and ending at his native city (which is one of these m cities). Can the traveller always fulfil his plans given that he can spend at most m doubloons if
- a [3] $m = 99$;
- a [3] $m = 100$?