International Mathematics

TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Spring 2014**

1 [**4**] Inspector Gadget has 36 stones with masses 1 gram, 2 grams, ..., 36 grams. Doctor Claw has a superglue such that one drop of it glues two stones together (thus two drops glue 3 stones together and so on). Doctor Claw wants to glue some stones so that in obtained set Inspector Gadget cannot choose one or more stones with the total mass 37 grams. Find the least number of drops needed for Doctor Claw to fulfil his task.

2 [**4**] In a convex quadrilateral $ABCD$ the diagonals are perpendicular. Points M and N are marked on sides AD and CD respectively. Prove that lines AC and MN are parallel given that angles ABN and CBM are right angles.

3 [**5**] Ali Baba and the 40 thieves want to cross Bosporus strait. They made a line so that any two people standing next to each other are friends. Ali Baba is the first; he is also a friend with the thief next to his neighbour. There is a single boat that can carry 2 or 3 people and these people must be friends. Can Ali Baba and the 40 thieves always cross the strait if a single person cannot sail?

4 [**5**] Positive integers a, b, c, d are pairwise coprime and satisfy the equation

$$ab + cd = ac - 10bd.$$

Prove that one can always choose three numbers among them such that one number equals the sum of two others.

5 [**5**] Park's paths go along sides and diagonals of the convex quadrilateral $ABCD$. Alex starts at A and hikes along $AB-BC-CD$. Ben hikes along AC; he leaves A simultaneously with Alex and arrives to C simultaneously with Alex. Chris hikes along BD; he leaves B at the same time as Alex passes B and arrives to D simultaneously with Alex. Can it happen that Ben and Chris arrive at point O of intersection of AC and BD at the same time? The speeds of the hikers are constant.